Learning Objective 6.4: I can implement an mPSK modulator and demodulator (BPSK, QPSK, 8PSK) and recover carrier and symbol timing.

Show your work. Write a **Documentation** line at the end (**None** if you did not collaborate).

1. **Bits per symbol.** A telemetry radio sends 25,000 symbols per second with RRC pulses of roll-off $\alpha = 0.25$. On a circle of radius 1, neighboring points sit 2.00 apart for BPSK, 1.41 apart for QPSK, and 0.77 apart for 8PSK.

   (a) Find the occupied bandwidth, and the bit rate with BPSK, QPSK, and 8PSK.

   B:

   bits:

   (b) The telemetry must carry 60 kb/s. Which of the three modulations can carry it at 25,000 symbols per second?

   (c) In words, explain what the radio gives up by moving from QPSK to 8PSK, and what you would watch on the receiver's constellation display as the signal weakens.

2. **The Gray map.** The L15 transmitter numbers the QPSK points by *phase index* $i = 0, 1, 2, 3$, counterclockwise from the point at $45°$. The Gray label of index i is $i \oplus (i \gg 1)$, the index XORed with itself shifted right one bit.

   (a) Write the 2-bit Gray label of each index. Then find the indices the transmitter sends for the bits `01 11 10 00`.

   labels: ____

   indices: ____

   (b) Noise moves a symbol sent as index 1 into the region of index 2. How many bits are wrong with the Gray labels? How many would be wrong if index i simply carried the binary number i?

   Gray: ____

   binary: ____

   (c) In words, explain why a wrong 8PSK symbol in a Gray-coded link usually corrupts only one of its three bits.

3. **Differential coding by hand.** The L15 Differential Encoder sends $e_n = (e_{n-1} + d_n) \bmod M$, where d_n is the phase index from the Gray map and e_{n-1} starts at 0. The decoder outputs $(r_n - r_{n-1}) \bmod M$, where r_n is the received index and r_{n-1} also starts at 0. The link uses QPSK, and the data indices are $d = 2, 1, 1, 3, 0$.

   (a) Find the five indices the encoder sends.

   (b) The receiver's Costas Loop locks 180° off, so every received index is 2 more than the one sent, mod 4. Decode the five received indices and compare them with d.

   (c) Now the loop locks correctly, but noise turns the third received index from 0 into 1. Decode the five indices and count the wrong symbols.

   (d) In words, use parts (b) and (c) to explain what differential coding does for a link and what it does to the errors in a noisy link.

4. **The frequency offset.** A frequency offset Δf turns the constellation by $360^\circ \times \Delta f / R_s$ every symbol. In Lesson 15, a QPSK Costas Loop at 4,000 symbols per second locked from a 200 Hz offset in under a second and false-locked from 450 Hz.

   (a) A 4 kbaud QPSK link has a 75 Hz offset. Find the rotation per symbol and the number of symbols in one full turn of the constellation.

   (b) In words, explain why a link between two separate radios has a frequency offset even when both are tuned to the same frequency, and why it needs a coarse frequency correction before its Costas Loop.

   (c) In words, explain why a receiver that sees the Lesson 15 constellation settle into clusters has not necessarily locked, and what you would check on its constellation, its frequency estimate, and its Received label to tell a true lock from a false one.

5. **QPSK or 8PSK?** A link must carry 12 kb/s in a channel 6.25 kHz wide, with RRC pulses of roll-off $\alpha = 0.35$ and differential coding. Lesson 15's Noise frame showed QPSK and 8PSK at the same noise amplitude, 0.8.

   (a) Find the symbol rate and the occupied bandwidth with QPSK and with 8PSK. Which fits the channel?

   QPSK:

   8PSK:

   (b) Describe what the receiver's constellation and Received label showed for each modulation at a noise amplitude of 0.8, and what that means for the 8PSK link.

   (c) In words, state which modulation you would choose and what you would check on the bench before committing to it.