Learning Objective 6.4: I can implement an mPSK modulator and demodulator (BPSK, QPSK, 8PSK) and recover carrier and symbol timing.

Show your work. Write a **Documentation** line at the end (**None** if you did not collaborate).

1. **Bits per symbol.** A telemetry radio sends 25,000 symbols per second with RRC pulses of roll-off $\alpha = 0.25$. On a circle of radius 1, neighboring points sit 2.00 apart for BPSK, 1.41 apart for QPSK, and 0.77 apart for 8PSK.

   (a) Find the occupied bandwidth, and the bit rate with BPSK, QPSK, and 8PSK.

   $$B = (1 + \alpha) R_s = 1.25 \times 25\ \text{kHz} = 31.25\ \text{kHz}$$

   The bandwidth does not depend on M. The bit rate is kR_s with $k = \log_2 M$: 25, 50, and 75 kb/s.

   B: 31.25 kHz

   bits: 25, 50, 75 kb/s

   (b) The telemetry must carry 60 kb/s. Which of the three modulations can carry it at 25,000 symbols per second?

   It needs at least $60/25 = 2.4$ bits per symbol. BPSK carries 1 and QPSK 2, so only 8PSK, with 3 bits and 75 kb/s, can carry it.

   8PSK

   (c) In words, explain what the radio gives up by moving from QPSK to 8PSK, and what you would watch on the receiver's constellation display as the signal weakens.

   8PSK sends 50% more bits in the same 31.25 kHz, but its points sit 0.77 apart instead of 1.41, so the same noise pushes far more symbols across to a neighbor. As the signal weakens, the noise clouds grow; the eight 8PSK clouds touch and merge into a ring while QPSK's four clouds are still separate, and the text starts to break up sooner.

2. **The Gray map.** The L15 transmitter numbers the QPSK points by *phase index* $i = 0, 1, 2, 3$, counterclockwise from the point at $45°$. The Gray label of index i is $i \oplus (i \gg 1)$, the index XORed with itself shifted right one bit.

   (a) Write the 2-bit Gray label of each index. Then find the indices the transmitter sends for the bits `01 11 10 00`.

   $$0 \oplus 0 = 00, \quad 1 \oplus 0 = 01$$
   $$2 \oplus 1 = 11, \quad 3 \oplus 1 = 10$$

   The labels are 00, 01, 11, 10. The transmitter's Map sends the index whose label matches each pair: `01` is 1, `11` is 2, `10` is 3, and `00` is 0, so it sends 1, 2, 3, 0.

   labels: 00, 01, 11, 10

   indices: 1, 2, 3, 0

   (b) Noise moves a symbol sent as index 1 into the region of index 2. How many bits are wrong with the Gray labels? How many would be wrong if index i simply carried the binary number i?

   Gray: `01` becomes `11`, one bit. Natural binary: `01` becomes `10`, two bits.

   Gray: 1

   binary: 2

   (c) In words, explain why a wrong 8PSK symbol in a Gray-coded link usually corrupts only one of its three bits.

   Almost every symbol error is a slip to one of the two neighbors, because the noise rarely reaches farther. With Gray labels a neighbor differs in one of the three bits, so each wrong symbol usually corrupts one bit out of three.

3. **Differential coding by hand.** The L15 Differential Encoder sends $e_n = (e_{n-1} + d_n) \bmod M$, where d_n is the phase index from the Gray map and e_{n-1} starts at 0. The decoder outputs $(r_n - r_{n-1}) \bmod M$, where r_n is the received index and r_{n-1} also starts at 0. The link uses QPSK, and the data indices are $d = 2, 1, 1, 3, 0$.

   (a) Find the five indices the encoder sends.

   $$e_0 = 0 + 2 = 2, \quad e_1 = 2 + 1 = 3$$
   $$e_2 = (3 + 1) \bmod 4 = 0$$
   $$e_3 = 0 + 3 = 3, \quad e_4 = 3 + 0 = 3$$

   2, 3, 0, 3, 3

   (b) The receiver's Costas Loop locks 180° off, so every received index is 2 more than the one sent, mod 4. Decode the five received indices and compare them with d.

   Received: $r = 0, 1, 2, 1, 1$. Decoded, with $r_{-1} = 0$:

   $$0 - 0 = 0,\ 1 - 0 = 1,\ 2 - 1 = 1,\ 1 - 2 \equiv 3,\ 1 - 1 = 0$$

   0, 1, 1, 3, 0

   The output is 0, 1, 1, 3, 0. Every symbol after the first is right; the first is wrong only because the decoder's starting reference was not rotated.

   (c) Now the loop locks correctly, but noise turns the third received index from 0 into 1. Decode the five indices and count the wrong symbols.

   Received: $r = 2, 3, 1, 3, 3$.

   $$2,\ 3 - 2 = 1,\ 1 - 3 \equiv 2,\ 3 - 1 = 2,\ 3 - 3 = 0$$

   2 wrong

   The output is 2, 1, 2, 2, 0: the third and fourth symbols are wrong, two errors from one wrong decision.

   (d) In words, use parts (b) and (c) to explain what differential coding does for a link and what it does to the errors in a noisy link.

   It makes the link work at any of the M lock positions, because a rotation adds the same amount to every index and the differences do not change. In exchange, every wrong decision corrupts two differences, the one into it and the one out of it, so one wrong decision corrupts two symbols of data.

4. **The frequency offset.** A frequency offset Δf turns the constellation by $360^\circ \times \Delta f / R_s$ every symbol. In Lesson 15, a QPSK Costas Loop at 4,000 symbols per second locked from a 200 Hz offset in under a second and false-locked from 450 Hz.

   (a) A 4 kbaud QPSK link has a 75 Hz offset. Find the rotation per symbol and the number of symbols in one full turn of the constellation.

$$360^\circ \times \frac{75}{4000} = 6.75^\circ$$

$$\frac{360^\circ}{6.75^\circ} = 53.3 \text{ symbols}$$

6.75°
53.3

   (b) In words, explain why a link between two separate radios has a frequency offset even when both are tuned to the same frequency, and why it needs a coarse frequency correction before its Costas Loop.

   Each radio makes its carrier from its own oscillator, and no two oscillators run at exactly the same rate, so the received carrier sits off from the receiver's by the difference. At a carrier of hundreds of megahertz or more, even a small error in each oscillator adds up to kilohertz, while a Costas Loop pulls in from only a few hundred hertz. A coarse correction, such as an FLL Band-Edge block or a frequency measured from a known preamble, brings the offset inside that range first.

   (c) In words, explain why a receiver that sees the Lesson 15 constellation settle into clusters has not necessarily locked, and what you would check on its constellation, its frequency estimate, and its Received label to tell a true lock from a false one.

   A loop can settle at the wrong frequency, a false lock: in Lesson 15, QPSK started 450 Hz off settled at -50 Hz. The points still form clusters, but twice as many as the constellation has, and no text arrives. A true lock shows M tight clusters at the transmitted angles, a frequency estimate equal to the offset that stays put, and lines arriving in the Received label.

5. **QPSK or 8PSK?** A link must carry 12 kb/s in a channel 6.25 kHz wide, with RRC pulses of roll-off $\alpha = 0.35$ and differential coding. Lesson 15's Noise frame showed QPSK and 8PSK at the same noise amplitude, 0.8.

   (a) Find the symbol rate and the occupied bandwidth with QPSK and with 8PSK. Which fits the channel?

   QPSK: $R_s = 12/2 = 6$ kbaud, $B = 1.35 \times 6 = 8.1$ kHz

   8PSK: $R_s = 12/3 = 4$ kbaud, $B = 1.35 \times 4 = 5.4$ kHz

   Only 8PSK fits in 6.25 kHz.

   QPSK: 8.1 kHz

   8PSK: 5.4 kHz

   (b) Describe what the receiver's constellation and Received label showed for each modulation at a noise amplitude of 0.8, and what that means for the 8PSK link.

   QPSK's four clouds nearly touched but stayed separate, and most lines arrived. 8PSK's eight clouds merged into a ring, and almost no line arrived intact. The 8PSK link fits the channel but needs a quieter channel, a stronger signal, than QPSK would at the same noise.

   (c) In words, state which modulation you would choose and what you would check on the bench before committing to it.

   8PSK, because QPSK does not fit the channel at 12 kb/s. On the bench, run the link at the weakest signal you expect and check that the eight clouds stay separate on the constellation display and the text arrives intact, with some room to spare. If they merge, the link needs more power or a slower bit rate.