Learning Objective 6.2: I can decode DTMF tones from a captured audio signal.

Show your work. Write a **Documentation** line at the end (**None** if you did not collaborate). The keypad grid: rows 697, 770, 852, and 941 Hz carry 1 2 3 A, 4 5 6 B, 7 8 9 C, and * 0 # D; columns are 1209, 1336, 1477, and 1633 Hz, left to right.

1. **Reading the grid.**

   (a) Which two tones does the '8' key send? The '#' key?

   '8' is row 3 and column 2: 852 Hz and 1336 Hz. '#' is row 4 and column 3: 941 Hz and 1477 Hz.

   8: 852, 1336 Hz

   #: 941, 1477 Hz

   (b) A receiver's bins show 941 Hz and 1209 Hz in one digit and 770 Hz and 1633 Hz in the next. Which two keys were pressed?

   941 and 1209 Hz is row 4, column 1: '*'. 770 and 1633 Hz is row 2, column 4: 'B'.

   *, B

   (c) In one block, the 697 Hz and 770 Hz bins are both strong and about equal, and so is the 1336 Hz bin. Which test of the L13 decoder rejects the block, and what could cause a signal like that?

   Test 2: the best row bin must beat the other three row bins by 6 dB, and here two row bins are equal. A keypad never sends two row tones at once, so the signal is something else: two keys pressed together, or speech or music with energy near both frequencies.

2. **Timing.** An auto-dialer sends each digit as 70 ms of tone followed by 15 ms of silence.

   (a) How long does one digit take, tone plus gap, and how long does an 11-digit number take, from the first tone to the end of the last one?

   Each digit takes $70 + 15 = 85$ ms.

   $$11 \times 85 - 15 = 920 \text{ ms}$$

digit:	85 ms
number:	0.92 s

   (b) Lesson 13 says DTMF tones last 40 ms or longer and gaps 20 ms or longer. Which does this dialer break? You set the same tone and gap on the sliders of `L13-dtmf-link.grc` and dial 7195550148. What would you expect to see in the Digits label?

   The 70 ms tones are fine, but the 15 ms gaps are shorter than 20 ms. A gap that short can fall between the decoder's blocks, so a repeated digit merges: some dialings read 71950148 or 719550148, with the three 5s counted as one or two.

3. **A shorter Goertzel block.** You want to try Goertzel blocks of $N = 136$ samples at $f_s = 8$ kHz in `L13-dtmf-rx.grc` instead of the lesson's 205.

   (a) Find the block time and the bin spacing, which is also the distance from a bin to its first null.

   $N/f_s = 136/8000 = 17$ ms
   $f_s/N = 8000/136 = 58.8$ Hz

   block: 17 ms

   spacing: 58.8 Hz

   (b) What do you change in the flowgraph, and what happens to the eight Goertzel blocks' Length and the two time sinks that plot the bins?

   Set the Variable `N` to 136. The eight Goertzel lengths and the `samp_rate/N` sample rate of the time sinks follow it, so nothing else changes; the sinks now get about 58.8 outputs a second.

   (c) Compared with $N = 205$, what do you expect the shorter block to do better, and what worse? Answer in words.

   Better: it responds faster, so it catches shorter tones, and its wider bins still decode a tone that is a little off frequency. Worse: its wider bins let more of the neighboring tone and more noise in, so the right bin wins by less, and short bursts of speech are more likely to look like a digit.

4. **Twist on the display.** A capture of the '9' key has a row tone of amplitude 0.20 and a column tone of amplitude 0.12.

   (a) In `L13-dtmf-rx.grc`, each stock Goertzel block feeds a Complex to Mag, which shows half a tone's amplitude. What heights do the 852 Hz and 1477 Hz traces reach?

   The 852 Hz trace reaches 0.10 and the 1477 Hz trace 0.06.

   852: 0.10

   1477: 0.06

   (b) Which tone arrives weaker? Which slider in `L13-dtmf-link.grc` makes the same kind of signal, and what happens to the Digits label as you push that slider further the same way?

   The column tone, 1477 Hz, is weaker. The Twist slider sets it; negative twist makes the column tone weaker. Digits keep decoding for a while, then stop at about -10 dB, the edge of test 3.

   (c) Why does the column tone often arrive weaker over a telephone line? Answer in one sentence.

   A telephone line loses more at high frequencies, and the column tones are the higher group.

5. **The decision rules.** The values below are one block's shares of its energy in each bin, as the L13 decoder sees them. Its tests: (1) the best row and column bins together hold at least 0.35; (2) each beats every other bin in its group by 6 dB, about four times the share; (3) the column share is between a tenth and four times the row share; (4) three blocks in a row agree.

Bin (Hz)	697	770	852	941	1209	1336	1477	1633
Block A	0.02	0.41	0.03	0.01	0.01	0.02	0.38	0.02
Block B	0.12	0.10	0.02	0.01	0.03	0.30	0.02	0.01

(a) Apply tests 1 to 3 to block A. Which digit does block A hold?

The best row bin is 770 Hz and the best column bin is 1477 Hz. Test 1: $0.41 + 0.38 = 0.79$, above 0.35. Test 2: 0.41 is far more than four times the next row bin, 0.03, and 0.38 far more than four times 0.02. Test 3: 0.38 is close to 0.41, well inside the limits. Block A holds '6'.

(b) Which test does block B fail?

The share is $0.12 + 0.30 = 0.42$, enough, but the best row bin, 0.12, is barely larger than the next, 0.10, nowhere near four times it, so block B fails test 2 and holds no digit.

(c) Block A alone does not produce a digit. What else must happen before the decoder reports '6', and what does that requirement reject?

Three blocks in a row, each starting 102 samples after the last, must all hold '6'. The tone must outlast about one 25.6 ms block, so short bursts of speech or noise that happen to pass the tests in one block do not count as digits.

6. **Noise.** You run `L13-dtmf-link.grc` and lower the SNR slider from 20 dB to 0 dB.

   (a) Describe what you see on the spectrum and in the Numbers label as the SNR comes down.

   The noise floor rises across the whole 0 to 4 kHz band until the two tones barely stand above it. The Numbers label stays at every number correct down to about 2 dB; near 1 dB some numbers come out short or wrong, and at 0 dB almost none arrive.

   (b) At 3 dB the tones barely show above the noise on the spectrum, yet every number still decodes. Explain in words why the decoder can still hear them.

   With bins 39 Hz apart, nulls 39 Hz either side, each Goertzel bin sees only a small slice of the noise spread across the 4 kHz band, while it still sees the whole of its tone. Inside its bin, the tone stands well above the noise.