Learning Objective 4.2: I can describe PAM theory and identify the pulse-shaping, symbol-rate, and bandwidth trade-offs.

Learning Objective 4.3: I can apply matched filtering and symbol-timing recovery to a received PAM signal.

Show your work. Write a **Documentation** line at the end (**None** if you did not collaborate).

1. **Constellations and Gray coding.**

   (a) Sketch the 4-PAM constellation on the I/Q plane and label its four points with a Gray code. Explain why a Gray code is worth using.

   Four points on the real axis at $-3, -1, +1, +3$, labeled 00, 01, 11, 10 from left to right. Noise almost always pushes a sample to a neighboring point, and neighbors differ in only one bit, so a wrong decision usually costs one bit instead of two.

   (b) 8-PSK places eight points evenly around a circle, three bits each. Label them with a Gray code, going around the circle.

   One answer, going counterclockwise from $0°$: 000, 001, 011, 010, 110, 111, 101, 100. Each point differs from both neighbors in one bit, including the wrap from 100 back to 000. Any labeling with that property is correct.

   (c) On a constellation sink, a 2-PAM signal shows two tight clouds at ± 1. The noise is turned up. Describe what changes on the display, and say when the receiver starts making wrong decisions.

   Each cloud spreads. While both clouds stay on their own side of the imaginary axis, the decision threshold, every decision is right; once the clouds spread far enough that some samples cross it, those samples are decided wrong.

2. **Fitting a channel.** A telemetry link must fit a 25 kHz channel and carry at least 32 kb/s. Use RRC pulse shaping, with bandwidth $B = (1+\alpha)R_s$.

   (a) With 2-PAM and $\alpha = 0.35$, find the largest symbol rate that fits and the bit rate it carries. Does it meet the requirement?

   $$R_s = \frac{B}{1+\alpha} = \frac{25\text{ kHz}}{1.35} = 18.5\text{ kbaud}$$

   One bit per symbol gives 18.5 kb/s, which is short of 32 kb/s.

   R_s: 18.5 kbaud

   meets it? no

   (b) Switch to 4-PAM at the same α. Which block setting changes, and what is the bit rate now? In words, what would you expect to see on the Symbol Sync output at the same noise as before, and why?

   The Chunks to Symbols table changes to four levels, and

   $$R_b = 2 \times 18.5 = 37.0\text{ kb/s}$$

   which meets the requirement. At the same power, four levels sit closer together than two, so the same noise spreads each band into its neighbors' thresholds sooner: more samples land past a threshold, and more decisions are wrong.

   R_b: 37.0 kb/s

   (c) Keep 2-PAM instead and send 32 kbaud. What roll-off would fit 25 kHz? Explain why that answer rules out this option.

   $$1+\alpha = \frac{25}{32} = 0.78 \quad \Rightarrow \quad \alpha = -0.22$$

   The roll-off cannot be negative: the narrowest any pulse can make the signal is $B = R_s = 32$ kHz at $\alpha = 0$, which is wider than the channel.

   impossible

3. **Setting up the transmit filter.** A flowgraph runs at `samp_rate` = 48 kHz and sends 2-PAM at 4.8 kbaud with $\alpha = 0.25$.

   (a) Find the samples per symbol and the occupied bandwidth.

   $\texttt{sps} = \dfrac{48\text{ kHz}}{4.8\text{ kbaud}} = 10, \qquad B = 1.25 \times 4.8 = 6.0\text{ kHz}$

   `sps`: 10

   B: 6.0 kHz

   (b) Write the `firdes.root_raised_cosine` call for the transmit taps, with a filter 11 symbols long, and name the block that uses them and its interpolation.

   `firdes.root_raised_cosine(10, 48000, 4800, 0.25, 110)`, in that order: gain `sps`, sample rate, symbol rate, α, and `11*sps` taps. The taps go in an Interpolating FIR Filter with interpolation 10.

   (c) A teammate wants 4.4 kbaud at the same sample rate. Explain why the flowgraph cannot do this as built, and give one way to fix it.

   $48/4.4 = 10.9$ samples per symbol, and the interpolating filter needs an integer interpolation. Change the sample rate to an integer multiple of the symbol rate, such as 44 kHz for $\texttt{sps} = 10$, or choose a symbol rate that divides 48 kHz, such as 4 kbaud.

   (d) The teammate shortens the filter to `2*sps` taps to save computation. What happens to the spectrum, and why?

   The filter truncates the pulse to two symbols. Cutting the pulse off abruptly in time spreads its spectrum, so the sidelobes outside $\pm B/2$ rise and the signal spills into the neighboring channels, which is the problem RRC shaping exists to solve.

4. **Reading an eye.** A QT GUI Eye Sink shows a 2-PAM signal at `sps` = 8 after the receive filter. The eye is widest open 3 samples after the start of each symbol period, where the traces cluster at ± 0.8 with a spread of ± 0.2 around each level.

   (a) Where should the receiver sample, and what is the eye opening there as a fraction of the ideal gap from -0.8 to $+0.8$?

   75%

   Sample 3 samples into each symbol. The innermost traces are at $0.8 - 0.2 = 0.6$ and -0.6, so the opening is

   $$\frac{0.6 - (-0.6)}{0.8 - (-0.8)} = \frac{1.2}{1.6} = 75\%$$

   (b) For each change below, say how the eye changes: (i) the noise standard deviation doubles; (ii) the receiver drops its filter; (iii) the receiver samples 4 samples away from the best instant.

   (i) The traces spread vertically, each band gets wider, and the opening between them shrinks. (ii) Noise across the whole sample bandwidth returns and the single transmit RRC leaves ISI, so the traces miss the levels and the eye nearly closes. (iii) Four samples is half a symbol, so the receiver samples between symbols. Where the symbol repeats, the sample stays near its level, but at a transition it lands near the zero crossing, and many of those decisions go wrong.

   (c) In words, explain why a smaller roll-off makes the receiver more sensitive to timing error.

   A smaller α gives a narrower spectrum, which means a longer pulse with larger tails. Away from the symbol instant those tails from many neighbors add up to more ISI, so the eye's waist is narrower and a given timing error closes it more.

5. **The matched filter.** The L9 matched-filter flowgraph runs at `samp_rate` = 32 kHz, `sps` = 8, and $\alpha = 0.35$.

   (a) The transmitter uses an RRC filter. What filter should the receiver use, and why does that choice also give zero ISI?

   The same RRC filter. The matched filter is the transmit pulse reversed in time, and an RRC pulse is symmetric, so it is its own match. The two RRC filters in cascade make a raised cosine, which is zero at every other symbol instant.

   (b) In the L9 matched-filter screenshot, at a noise amplitude of 0.6, the eye before the filter is closed and the eye after it is open. The signal occupies about ± 2.7 kHz of the 32 kHz the samples represent. Explain in words what the filter did to make the difference.

   The noise is spread across the whole 32 kHz, but the signal sits in only a few kilohertz of it. The matched filter passes the signal's band and blocks the noise everywhere else, so far less noise is left at the sampling instant, the traces converge near ± 1, and the eye opens.

   (c) A classmate puts a full raised-cosine filter at the transmitter and no filter at the receiver. The eye at the receiver has zero ISI. Explain what is still wrong with this design.

   The receiver passes all the noise across the 32 kHz the samples represent, instead of only the signal's band, so the eye is noisier and more decisions are wrong at the same transmit power. Zero ISI is necessary but not sufficient; the RRC split gets zero ISI and the best noise rejection together.

6. **Timing recovery.** A 2-PAM receiver at 4 kbaud uses Keep 1 in N to pick one sample per symbol, starting at the center of the eye. Its sample clock is 50 ppm fast relative to the transmitter's.

   (a) How many symbols, and how many seconds, until the sampling point has drifted half a symbol, to the zero crossings?

   Each symbol the sampling point moves 50×10^{-6} of a symbol, so

   $$N = \frac{0.5}{50 \times 10^{-6}} = 10{,}000 \text{ symbols}$$
   $$t = \frac{10{,}000}{4000} = 2.5 \text{ s}$$

   symbols: 10,000

   time: 2.5 s

   (b) You replace Keep 1 in N with a Symbol Sync block. A teammate then puts a Keep 1 in N block in front of it to save computation, so Symbol Sync receives one sample per symbol, and sets its samples per symbol to 1. Explain why timing recovery no longer works, and say where the one-sample-per-symbol step belongs.

   The Gardner detector needs a sample at each symbol instant and one halfway between, so it needs at least 2 samples per symbol; with 1 it has no midpoint sample and cannot tell early from late. Symbol Sync should receive all 8 samples per symbol from the matched filter; its own output, at 1 sample per symbol, is the one-sample-per-symbol step.

   (c) The Symbol Sync output jitters around ± 1 even with little noise. Which parameter should change, in which direction, and what is the trade-off?

   Lower the loop bandwidth. A narrower loop averages the timing error over more symbols, so the sampling instant jitters less, but the loop locks more slowly and follows a drifting clock less closely.

Documentation: _____