Learning Objective 4.1: I can describe FSK modulation and implement an FSK transmitter and receiver in GNU Radio.

Show your work. Write a **Documentation** line at the end (**None** if you did not collaborate).

1. **Transmitter settings.** A flowgraph samples at $f_s = 48$ kHz and sends FSK at 1200 baud with a deviation of $\Delta f = 600$ Hz, one bit per symbol.

   (a) Find the samples per symbol to enter in the Repeat block.

   $$\text{sps} = \frac{f_s}{R_s} = \frac{48000}{1200} = 40$$

   40

   (b) Find the sensitivity to enter in the Frequency Mod block.

   $$k = \frac{2\pi\Delta f}{f_s} = \frac{2\pi \times 600}{48000} = 0.0785 \text{ rad/sample}$$

   0.0785

   (c) Find the modulation index and estimate the bandwidth with Carson's rule.

   $$h = \frac{2\Delta f}{R_s} = \frac{1200}{1200} = 1$$
   $$B \approx 2(\Delta f + R_s) = 2(600 + 1200) = 3.6 \text{ kHz}$$

   h = 1

   B = 3.6 kHz

   (d) A classmate builds the same transmitter from two Signal Sources at $+600$ and -600 Hz and a selector that switches between them at each bit. Explain why Frequency Mod gives a more compact spectrum.

   The two sources run independently, so at each switch the phase jumps from wherever one source is to wherever the other is. Those jumps are abrupt changes in the signal, and abrupt changes spread power across a wide band. Frequency Mod adds a phase step every sample, so the phase is continuous across each bit boundary and the spectrum stays compact.

2. **Receiver settings.** The receiver for the link of Question 1 follows the L8 flowgraph: Low Pass Filter, Quadrature Demod, Moving Average, Symbol Sync, and Binary Slicer.

   (a) Find the Quadrature Demod gain that puts the two tones at $+1$ and -1.

   $$g = \frac{f_s}{2\pi\Delta f} = \frac{48000}{2\pi \times 600} = 12.7$$

   12.7

   (b) Find the low-pass filter cutoff that passes the Carson bandwidth.

   The filter passes $-f_c$ to $+f_c$, so the cutoff is half of B:

   $$f_c = \Delta f + R_s = 600 + 1200 = 1800 \text{ Hz}$$

   1800 Hz

   (c) A classmate leaves the Quadrature Demod gain at 1. Find the levels the two tones produce, and explain whether the slicer still decides correctly.

   With $g = 1$ the output is the phase step itself, $\pm 2\pi\Delta f / f_s = \pm 0.0785$. The slicer compares with zero, and both levels are still on the correct side of zero, so the bits come out correctly. Only the scale is wrong: the eye diagram and any plot set for ± 1 show a nearly flat line.

   (d) The classmate also sets the filter cutoff to 8 kHz. Explain what changes on the eye diagram, and what happens to the noise voltage at which the text first breaks.

   The signal still fits through the filter, but the filter now passes far more noise, everything from -8 to $+8$ kHz instead of -1.8 to $+1.8$ kHz. The eye's traces are thicker at the same noise setting, and the text breaks at a lower noise voltage than with the 1800 Hz cutoff.

3. **Noise.** The link of Questions 1 and 2 ($f_s = 48$ kHz, 1200 baud, $\Delta f = 600$ Hz, cutoff 1800 Hz) runs through a Channel Model with a noise voltage slider. With the slider low, the eye is wide open and every line of text arrives.

   (a) The noise slider is raised a long way. Describe what you expect to see on the spectrum, on the eye diagram, and in the received text.

   The tones stay where they are and the floor rises toward them. The eye's traces thicken, and now and then a sharp spike, an FM click, crosses zero. Lines start to arrive with wrong characters, and with still more noise the receiver stops finding the access code.

   (b) A spike crosses zero on the eye diagram, but away from the sampling instant. Explain whether it causes a bit error.

   No. The Binary Slicer only sees the one sample per bit that Symbol Sync picks at the peak of each bit. A spike that crosses zero between those instants never reaches the slicer; only one at a sampling instant flips a bit.

   (c) The link raises its symbol rate to 1600 baud and keeps $\Delta f = 600$ Hz. Find the new samples per symbol, bandwidth, and filter cutoff.

   $\text{sps} = 48000/1600 = 30$

   $B = 2(600 + 1600) = 4.4\,\text{kHz}, \qquad f_c = 2.2\,\text{kHz}$

   sps = 30

   B = 4.4 kHz

   f_c = 2.2 kHz

   (d) Explain whether the 1600-baud link's text breaks at a higher or a lower noise setting than the 1200-baud link's.

   Lower. The wider filter lets in more of the noise, so the eye is noisier at the same slider setting and the text breaks sooner.

   (e) On an FSK link the text goes from clean to almost no packets over a small range of the noise slider. Lesson 2 designs a link with margin. Explain, from that observation, why margin matters for FSK.

   A link that only just works at its design distance fails completely when the signal fades a little. Margin keeps the link on the clean side when the path loss changes.

4. **An FSK remote on a spectrogram.** An RTL-SDR capture, viewed as a spectrogram in URH, shows a burst with two tracks, at 433.900 and 433.940 MHz. In the alternating preamble, the signal stays on each track for 0.417 ms.

   (a) Find the center frequency, the deviation, and the symbol rate.

$$f_c = \frac{433.900 + 433.940}{2} = 433.920 \text{ MHz}$$
$$\Delta f = \frac{40 \text{ kHz}}{2} = 20 \text{ kHz}$$
$$R_s = \frac{1}{0.417 \text{ ms}} = 2400 \text{ baud}$$

   f_c = 433.920 MHz

   Δf = 20 kHz

   R_s = 2400 baud

   (b) The remote's crystal is accurate to 10 ppm. Find the largest frequency offset at 433.92 MHz.

$$10 \times 10^{-6} \times 433.92 \times 10^{6} = 4.3 \text{ kHz}$$

   4.3 kHz

   (c) An offset adds $f_{\text{off}}/\Delta f$ to both demodulator levels. Find the remote's two levels with a 4.3 kHz offset, and say whether the slicer still decides correctly. Then do the same for a 433.92 MHz link built with $\Delta f = 2$ kHz.

   Remote: $4.3/20 = 0.22$, so the levels are $+1.22$ and -0.78. Both are on the correct side of zero, so the slicer still decides correctly; the large deviation is what makes a 10 ppm crystal acceptable. With $\Delta f = 2$ kHz: $4.3/2 = 2.15$, so the levels would be $+3.15$ and $+1.15$, both above zero, and every 0 would come out as a 1. In practice the tones also move off the receiver's filter, and the link fails.

   (d) Name one change to the remote's receiver that would put its levels back at ± 1.

   Subtract the average of the demodulator output, for example with a DC Blocker after the Quadrature Demod, because the offset appears as a constant $f_{\text{off}}/\Delta f$ added to both levels. Measuring the offset and shifting the signal back in frequency before the filter also works.

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