Learning Objective 3.1: I can describe DSB-AM, DSB-SC, and SSB and identify each on a spectrum.
Learning Objective 3.2: I can implement an AM modulator and demodulator in GNU Radio and verify demodulated audio against the source.

Show your work. Write a **Documentation** line at the end (**None** if you did not collaborate).

1. **Reading a spectrum.** A 1 kHz tone modulates a 100 kHz carrier.

   (a) List the frequencies of the spectral lines for DSB-AM, DSB-SC, USB, and LSB.

Signal	Lines (kHz)	Bandwidth (kHz)
DSB-AM		
DSB-SC		
USB		
LSB		

   DSB-AM: 99, 100, and 101 kHz, 2 kHz wide. DSB-SC: 99 and 101 kHz, 2 kHz wide. USB: 101 kHz only, 1 kHz of message bandwidth. LSB: 99 kHz only, 1 kHz of message bandwidth.

   (b) A spectrum display shows two lines of equal height at 14.997 and 15.003 MHz and nothing between them. Name the modulation, the carrier frequency, and the message tone.

   DSB-SC: the lines are symmetric about 15.000 MHz, so that is the carrier, and there is no carrier line. The tone is $15.003 - 15.000 = 3$ kHz.

   (c) A waterfall shows a voice signal from 7.1970 to 7.1997 MHz that comes and goes with speech, with no steady line beside it. Voice runs from 300 Hz to 3 kHz. Name the modulation and the frequency a receiver must tune to.

   SSB, because it is about 2.7 kHz wide, one-sided, and carrierless. The audio from 300 Hz to 3 kHz lies *below* the carrier, so it is LSB with the carrier at

$$7.1970 + 0.0030 = 7.1997 + 0.0003 = 7.2000 \text{ MHz}$$

   LSB below 10 MHz is the amateur convention.

2. **DSB-AM.** A transmitter sends $s(t) = [1 + m\cos(2\pi f_m t)]\cos(2\pi f_c t)$, with m set by a slider.

   (a) On the flowgraph's frequency sink you drag m from 0.5 up to 1. Say which lines change and which do not.

   The two sideband lines grow, because m scales the message before it reaches the carrier. The carrier line at f_c stays the same height, because it comes from the constant 1, which m does not touch.

   (b) The carrier is the tallest line on the display. Explain why it carries no information, and why AM broadcasting keeps it anyway.

   The carrier line is the same whatever the message is, so it tells the listener nothing. It is there so the receiver can be an envelope detector, a diode and a capacitor, with no oscillator of its own to match.

   (c) A station carries audio up to 5 kHz. Find its bandwidth.

   $2W = 2 \times 5 = 10$ kHz

   10 kHz

   (d) The operator raises m to 1.2. Explain in words what an envelope detector now outputs, and why it no longer matches the message.

   At the message's troughs, $1 + 1.2\cos(2\pi f_m t)$ goes as low as -0.2. The carrier's amplitude cannot be negative, so the envelope is the absolute value, and the detector outputs $|1 + m\cos(2\pi f_m t)|$: the troughs fold back up. The audio distorts, and the new frequencies spread the signal outside its channel.

3. **The envelope detector flowgraph.** The receiver in `L06-dsb-am.grc` is Abs, then a low-pass filter, then Multiply Const. The flowgraph runs at 48 kHz with a 10 kHz carrier and a message up to 2.5 kHz.

   (a) The low-pass filter has a 3 kHz cutoff and a 1 kHz transition. Explain why that works: what must it pass, and what must it stop?

   It must pass the envelope, which holds the message up to 2.5 kHz, and stop the higher-frequency content that Abs leaves behind. A 3 kHz cutoff with a 1 kHz transition passes everything up to about 2.5 kHz, which keeps both tones, and stops everything above about 3.5 kHz.

   (b) Explain the Multiply Const of $\pi/2$ after the filter.

   Averaging the rectified carrier leaves $2/\pi$ of its peak, so the filter's output is too small by that factor. Multiplying by $\pi/2$ restores the envelope's level so we can compare it with $1 + m\,x(t)$ on the time sink.

   (c) The time sink shows the detector output lagging the envelope by 57 samples. Find the lag in milliseconds, and name the block that causes it.

   1.19 ms

   $$\frac{57}{48000} = 1.19\,\text{ms}$$

   The low-pass filter delays its output by half its length. The flowgraph's Delay block delays the reference by the same 57 samples so the traces line up.

   (d) The flowgraph has an Audio Sink and no Throttle. Explain why.

   The sound card takes exactly 48,000 samples per second, so the Audio Sink paces the flowgraph the way an SDR does. A Throttle would add a second clock competing with the sound card's and cause audio dropouts.

4. **Coherent and product detectors.**

(a) In `L06-dsb-sc.grc` you drag the LO phase slider from 0° to 90° with the audio on. Describe what the coherent receiver's trace on the time sink and the audio do.

The red trace shrinks toward zero as the phase error grows, and the tones get quieter. At 90° the trace is flat at zero and the audio is silent. The pitch never changes; only the level does.

(b) Explain why a DSB-SC receiver on the air needs carrier recovery, and how FM stereo provides it for its DSB-SC subcarrier.

The receiver's LO must match the carrier in frequency and phase, and DSB-SC transmits no carrier to lock to; at 90° the output vanishes. FM stereo sends a 19 kHz pilot tone, and the receiver doubles it to rebuild the 38 kHz LO in the right phase.

(c) A USB signal carries a 1 kHz tone. The product detector's LO sits 150 Hz above the carrier. Find the output tone. Repeat for LSB.

The USB tone is at $f_c + 1000$; mixing with $f_c + 150$ leaves $1000 - 150 = 850$ Hz. The LSB tone is at $f_c - 1000$, which leaves $1000 + 150 = 1150$ Hz.

USB: 850 Hz

LSB: 1150 Hz

(d) The SSB flowgraph's Delay block holds the message back $(129 - 1)/2 + 57$ samples before comparing it with the receiver's output. Find the delay and explain each term.

$$64 + 57 = 121 \text{ samples}$$

The 129-tap Hilbert filter delays the message by 64 samples, half its length, and the receiver's low-pass filter adds 57 more.

121 samples

Documentation: _____