Learning Objective 1.3: I can describe signal-space concepts (basis functions, constellations, decision regions) and apply them to a digital modulation scheme.

Show your work. Write a **Documentation** line at the end (**None** if you did not collaborate).

1. **Three families of modulation.** A link must carry 1 Mb/s.

   (a) For each family, say what the transmitter changes from one symbol to the next: ASK/PAM, PSK, and FSK.

   ASK and PAM change the amplitude, PSK changes the phase of the carrier, and FSK changes the frequency.

   (b) How many bits per symbol do 4-PAM, 8-PSK, and 16-PSK carry?

   $\log_2 M$: 4-PAM carries 2, 8-PSK carries 3, and 16-PSK carries 4.

4-PAM:	2 bits
8-PSK:	3 bits
16-PSK:	4 bits

   (c) Find the symbol rate the link needs with BPSK and with 8-PSK. This is the number a modulator block's samples-per-symbol setting is built around.

   Divide the bit rate by the bits per symbol: BPSK needs $10^6/1 = 1$ Msym/s, and 8-PSK needs $10^6/3 = 333$ ksym/s.

BPSK:	1 Msym/s
8-PSK:	333 ksym/s

2. **Binary PAM and noise.** A binary PAM receiver samples each pulse once. A 1 is sent as $+1$ V and a 0 as -1 V.

   (a) State the decision rule, and sketch the two points and the threshold on a line.

   Decide 1 if the sample is above 0 V and 0 if it is below. The threshold sits at 0 V, halfway between the two points.

   (b) Decide each of these received samples: $+0.83$, -1.20, $+0.07$, -0.31 V. The bits sent were 1, 0, 0, 0. Which sample did the receiver decide wrong, and why?

   1, 0, 1, 0. The third sample was sent as -1 V, but the noise added more than 1 V and carried it past the threshold, so the receiver decided it was a 1.

   (c) The SDR display shows the SNR falling from 14 dB to 4 dB. Describe what happens to the received samples, and what you would see on a time sink of the slicer's output.

   Each point's spread of samples widens until the spreads meet at 0 V, and some samples land on the wrong side. The threshold does not move. On the time sink, the slicer's output, which was a clean copy of the sent bits, flips to the wrong value from time to time.

   (d) Name two changes that would keep more samples on the right side of the threshold.

   Raise the SNR (more transmit power, a better antenna, a lower-noise first stage, a narrower receiver filter), or move the two points farther apart for the same power, which antipodal points already do.

3. **Antipodal, orthogonal, and constellations.**

   (a) Sketch binary PAM (antipodal) and binary FSK (orthogonal) at the same energy, with the boundary between the two points of each. Which pair is farther apart?

   Antipodal: points at -1 and $+1$ on the I axis, boundary on the Q axis. Orthogonal: one point on each axis, at $(1, 0)$ and $(0, 1)$, boundary on the diagonal. The antipodal points are farther apart.

   (b) Both are sent through the same noise. Which one puts more samples in the wrong region, and why?

   The orthogonal pair. The clouds are the same size, but its points sit closer to their boundary, so more samples stray across.

   (c) Sketch QPSK and 4-PAM at the same average energy, mark the decision regions of each, and explain why a radio uses QPSK rather than 4-PAM on a carrier.

   QPSK has one point per quadrant, and its regions are the quadrants. 4-PAM has four points on the I axis, and its regions are strips separated by thresholds halfway between neighbors. Both carry two bits, but QPSK uses both axes and keeps its points farther apart for the same power, so the same noise puts fewer samples in the wrong region.

4. **Gray coding.** An 8-PSK modem places its points every 45° around a circle.

   (a) Label the eight points with a Gray code, starting from 000 at 0°.

   Going counterclockwise: 000, 001, 011, 010, 110, 111, 101, 100. Each label differs from both of its neighbors, including 100 and 000, in one bit.

   (b) With natural binary labels instead (000, 001, 010, ..., 111), how many bits does a slip from the point labeled 011 to its neighbor 100 corrupt?

   011 and 100 differ in all three positions, so all three bits are wrong.

   3 bits

   (c) Explain in one or two sentences why a Gray code helps, given where noise usually pushes a sample.

   Noise almost always pushes a sample into a neighbor's region, not across the constellation. With a Gray code every neighbor differs by one bit, so that slip costs one bit instead of up to three.

5. **Choosing a modulation.** In the constellation explorer, QPSK at an SNR of 12 dB shows almost no samples in the wrong region. Switching to 8-PSK at the same SNR turns a steady trickle of samples red.

   (a) Explain what changed on the plot, using the decision regions.

   8-PSK puts twice as many points on the same circle, so they sit closer together and the regions are narrower. The clouds are the same size, so they now reach the boundaries.

   (b) What does 8-PSK give you in exchange?

   Three bits per symbol instead of two, so 50% more data at the same symbol rate and bandwidth.

   (c) A link's SDR display shows only a little more SNR than QPSK needed to stay clean. Which modulation do you choose, and what would make you change your mind?

   QPSK: its clouds stay inside their regions with the SNR the link has, and the margin covers a fade. 8-PSK becomes the choice only if the link gains SNR, from more power, better antennas, or a shorter range, enough that its clouds stay clean too.

6. **Reading a constellation.** Three QPSK receivers each show one problem on a constellation sink. Name the cause of each.

   (a) The four points sit in the middle of their quadrants, but each is a round cloud about a third of the way to the boundaries.

   Noise. The cloud's size shows how good the SNR is; nothing else is wrong.

   (b) Four tight clouds, each rotated 30° counterclockwise from where it belongs, standing still.

   A fixed phase offset between the transmitter and receiver oscillators.

   (c) The points are smeared into a ring of constant radius.

   A frequency offset: the phase error grows continuously, so the constellation rotates through every angle during the capture.

   (d) Explain why the constellation in part (b) would put every symbol in the wrong region if the rotation passed 45°, even with no noise.

   The QPSK decision boundaries are the axes, 45° from each point. A rotation of 45° puts every point on a boundary, and anything past it puts every point in a neighbor's region, so the receiver must correct the phase before it decides.

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