ECE448 Spring 2025

Lesson 03

Signal Space



Nominal Schedule

Lesson	Topics	Notes
1	Course Overview & Introduction	
2	Noise Figure & Link Budget	
3	Signal Space	
4	PAM Theory	
5	ASCII/PAM	**Theory Project Due**
6	ASCII/PAM	
7	ASCII/PAM	
8	GRC: OOT Modules	
9	GRC: Custom Blocks	**PAM Project Due**
10	GRC: Custom Blocks	
11	CW	
12	CW	**Custom Block Project due**
13	CW	
14	DTMF	
15	DTMF	**CW Project Due**
16	Flex Day	**Final Project Intro**
17	FHSS - Tx	
18	FHSS - Rx	**DTMF Project Due**
19	Midterm (in-class portion)	
20	Flex day	

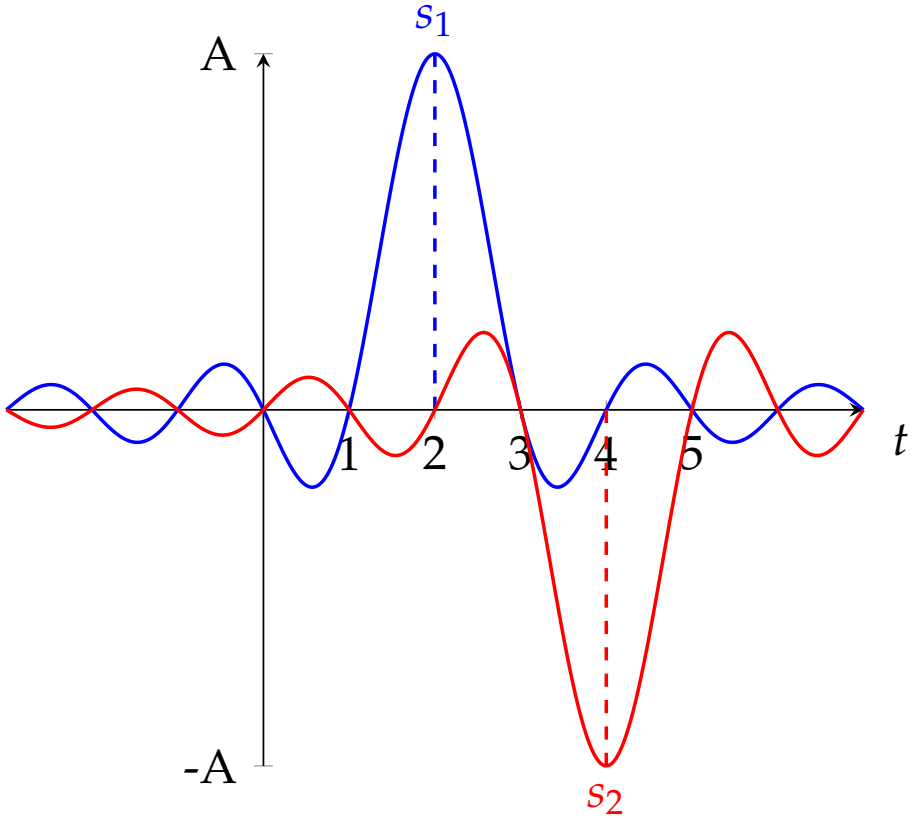
Lesson 3 Learning Objectives

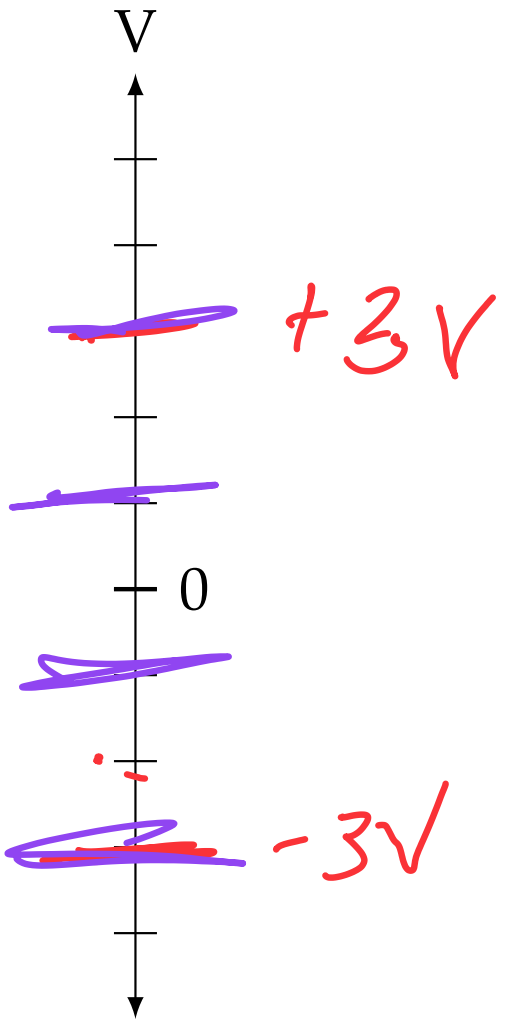
1.1 Understand and articulate the difference between mFSK, mASK, PAM, and mPSK signals.

1.2 Calculate the probability of error for each.

1.3 Draw the constellation diagram for a given modulation technique and

Binary PAM





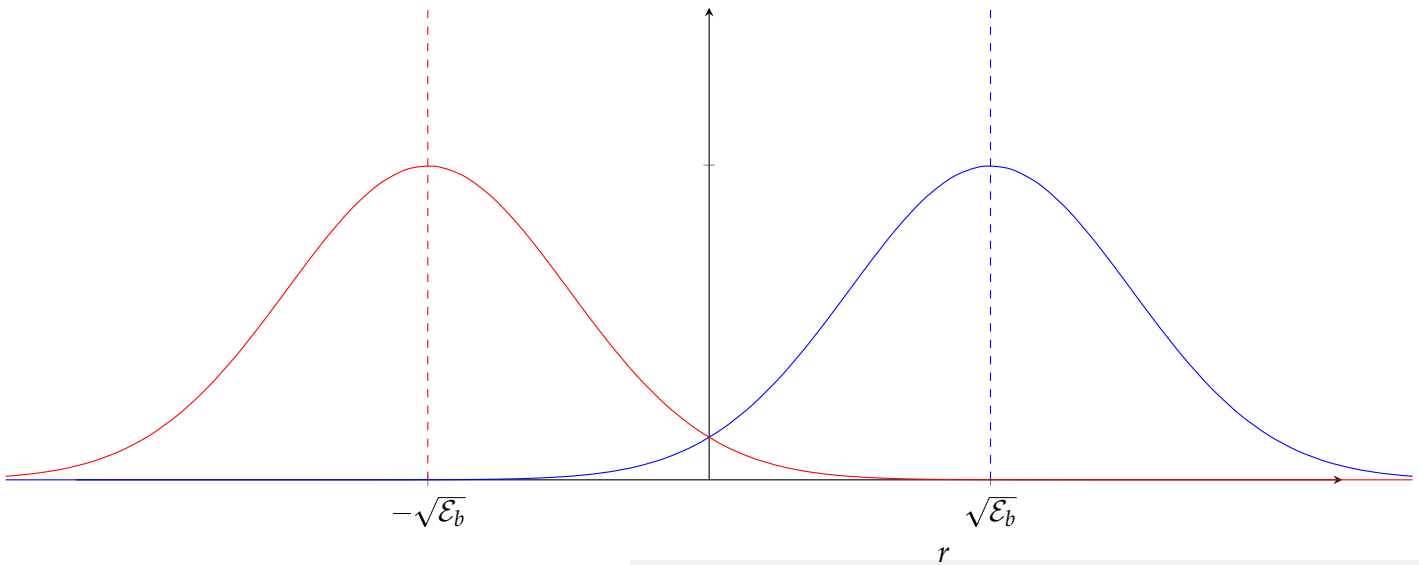


PAM Probability of Error P_b

- Assume the energy per bit in a PAM pulse is _____.
- In AWGN channel, we have the received signal as: $r = s_k + n$
- The conditional probability of receiving each pulse:

$$p(r|s_1) = \frac{1}{\sqrt{\pi N_0}} e^{-\left(r-\sqrt{\mathcal{E}_b}\right)^2/N_0}$$

$$p(r|s_2) = \frac{1}{\sqrt{\pi N_0}} e^{-\left(r+\sqrt{\mathcal{E}_b}\right)^2/N_0}$$



PAM P_b

Given a decision threshold of 0, the probability of an error for s_1 tx'd is:

$$P(e|s_1) = \int_{-\infty}^{0} p(r|s_1)dr = \frac{1}{\sqrt{\pi N_0}} \int_{-\infty}^{0} e^{-\left(r-\sqrt{\mathcal{E}_b}\right)^2/N_0} dr$$

PAM P_b

Given a decision threshold of 0, the probability of an error for s_1 tx'd is:

$$P(e|s_1) = \int_{-\infty}^{0} p(r|s_1)dr = \frac{1}{\sqrt{\pi N_0}} \int_{-\infty}^{0} e^{-\left(r-\sqrt{\mathcal{E}_b}\right)^2/N_0} dr$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0} e^{-\left(r-\sqrt{\mathcal{E}_b}\right)^2/N_0} dr = Q\left(\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right)$$

PAM P_b

Given a decision threshold of 0, the probability of an error for s_1 tx'd is:

$$P(e|s_1) = \int_{-\infty}^{0} p(r|s_1)dr = \frac{1}{\sqrt{\pi N_0}} \int_{-\infty}^{0} e^{-\left(r-\sqrt{\mathcal{E}_b}\right)^2/N_0} dr$$
$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0} e^{-\left(r-\sqrt{\mathcal{E}_b}\right)^2/N_0} dr = Q\left(\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right)$$

Since both signals are equally likely...

PAM Probability of Error

$$P_b = \frac{1}{2}P(e|s_1) + \frac{1}{2}P(e|s_2) = Q\left(\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right)$$

PAM P_b

The energy "distance" between the two signals is $d_{12} = 2\sqrt{\mathcal{E}_b}$, implying:

$$\mathcal{E}_b = \frac{d_{12}^2}{4}$$

PAM P_b

The energy "distance" between the two signals is $d_{12} = 2\sqrt{\mathcal{E}_b}$, implying:

$$\mathcal{E}_b = \frac{d_{12}^2}{4}$$

Leading to the big result....

PAM Probability of Error

$$P_b = Q\left(\sqrt{\frac{d_{12}^2}{2N_0}}\right)$$

Orthogonal P_b

PAM signals are antipodal, but what if the signals are orthogonal?

$$s_1 = \left(\sqrt{\mathcal{E}_b}, 0\right)$$
$$s_2 = \left(0, \sqrt{\mathcal{E}_b}\right)$$

Orthogonal Signals P_b

$$P_b = Q\left(\sqrt{\frac{\mathcal{E}_b}{N_0}}\right)$$

$m-$ARY PAM P_b

$m-$ary PAM signals

$$s_m = \sqrt{\mathcal{E}_g} A_m$$

$m-$ary PAM

$$P_m = \frac{2(M-1)}{M} Q\left(\sqrt{\frac{6\ln(M)\,\mathcal{E}_{b,\text{avg}}}{(M^2-1)\,N_0}}\right)$$

$m-$ARY PAM P_b (BAND-LIMITED)

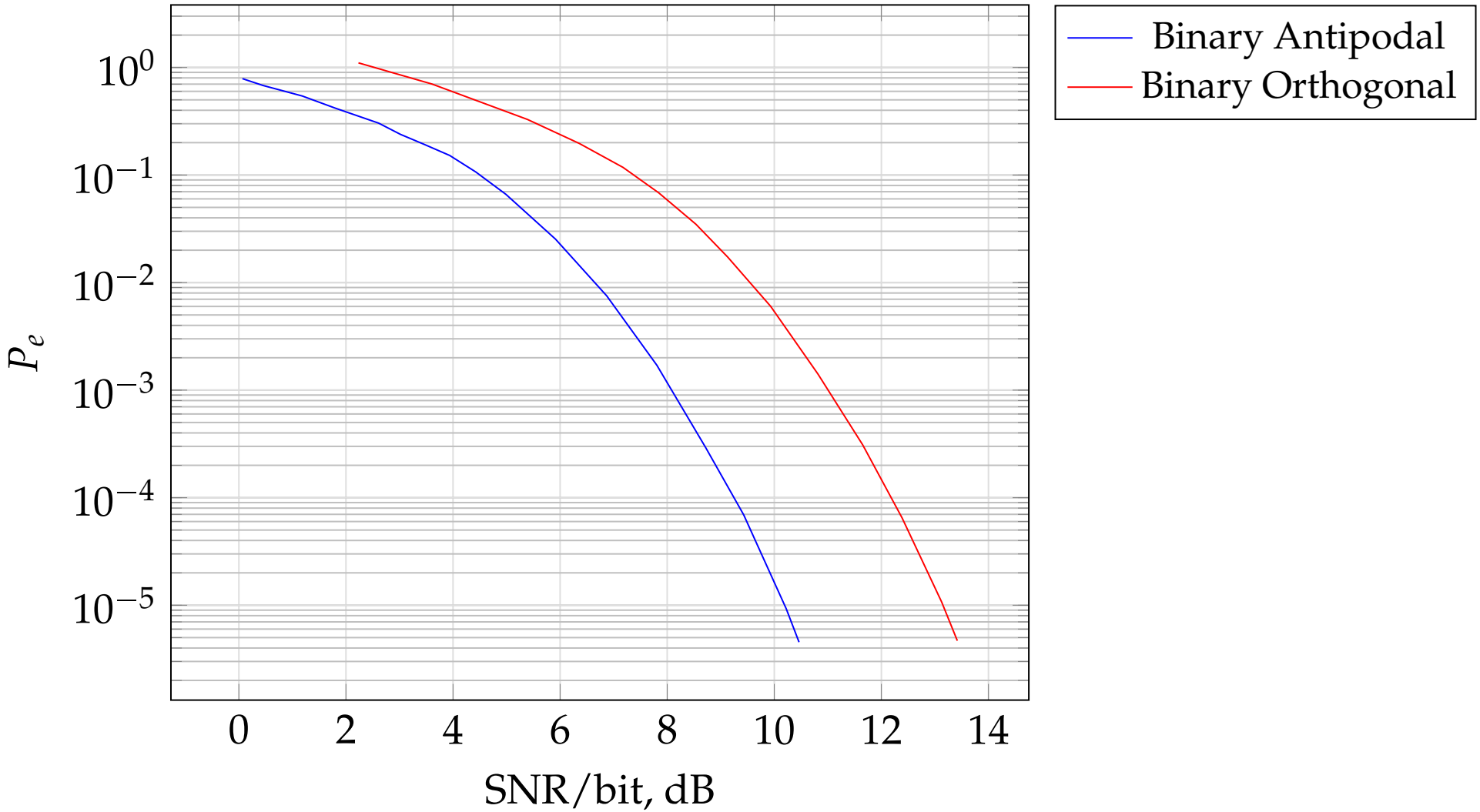
$m-$ary PAM signals

$$s_m = \sqrt{\mathcal{E}_g} A_m$$

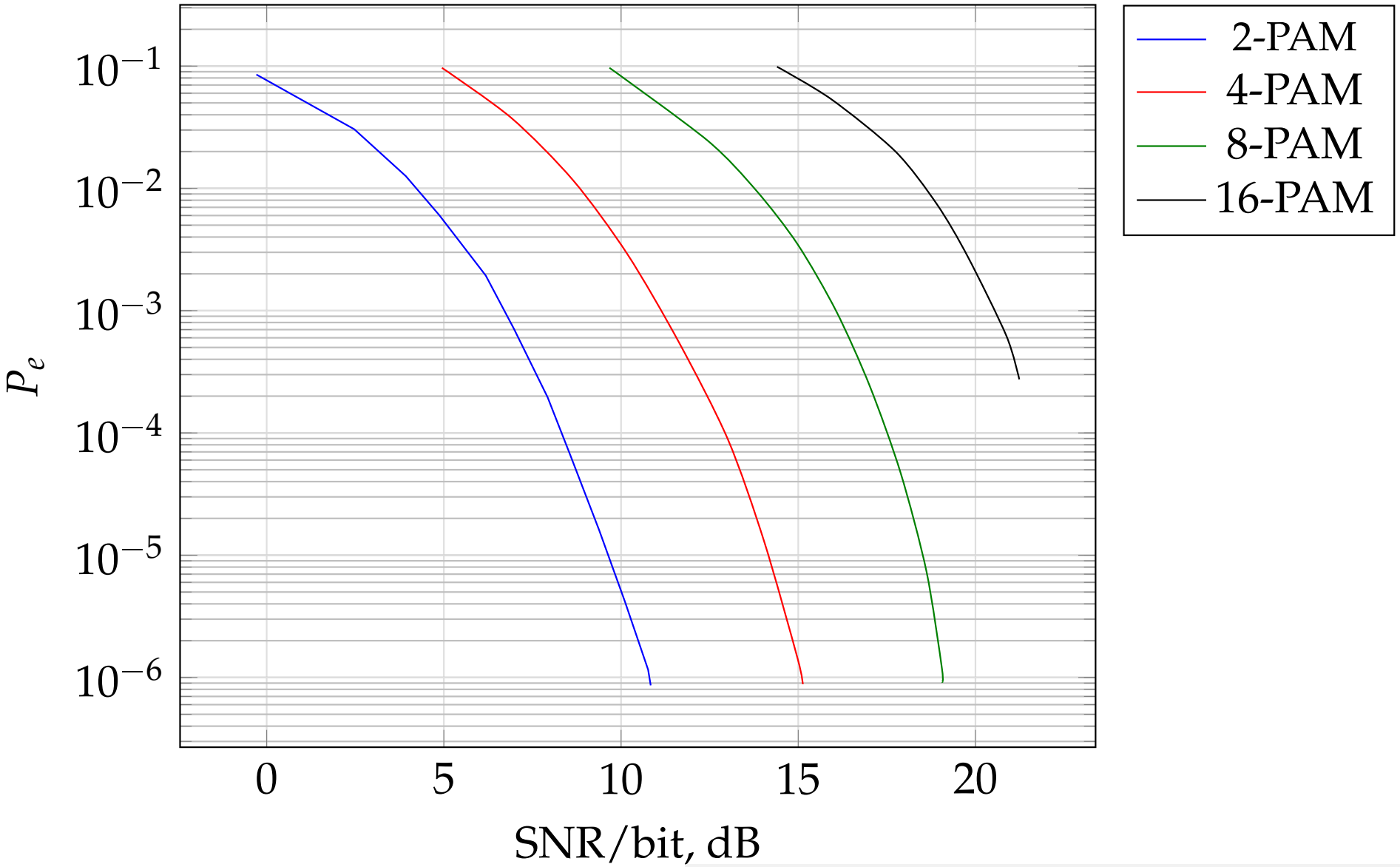
$m-$ary PAM

$$P_m = \frac{2(M-1)}{M} Q\left(\sqrt{\frac{6\mathcal{E}_{b,\text{avg}}}{(M^2-1)N_0}}\right)$$

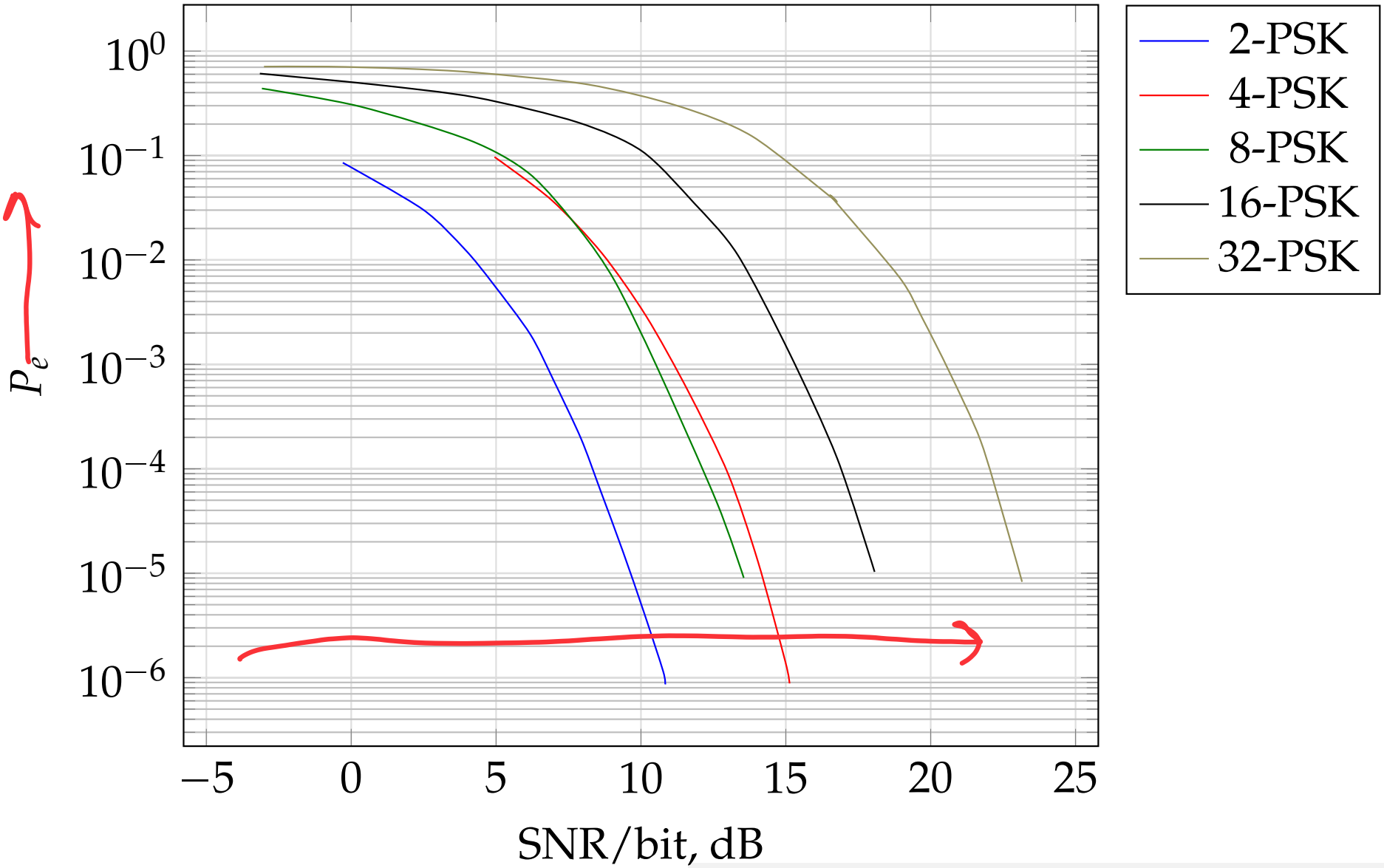
Binary Signals



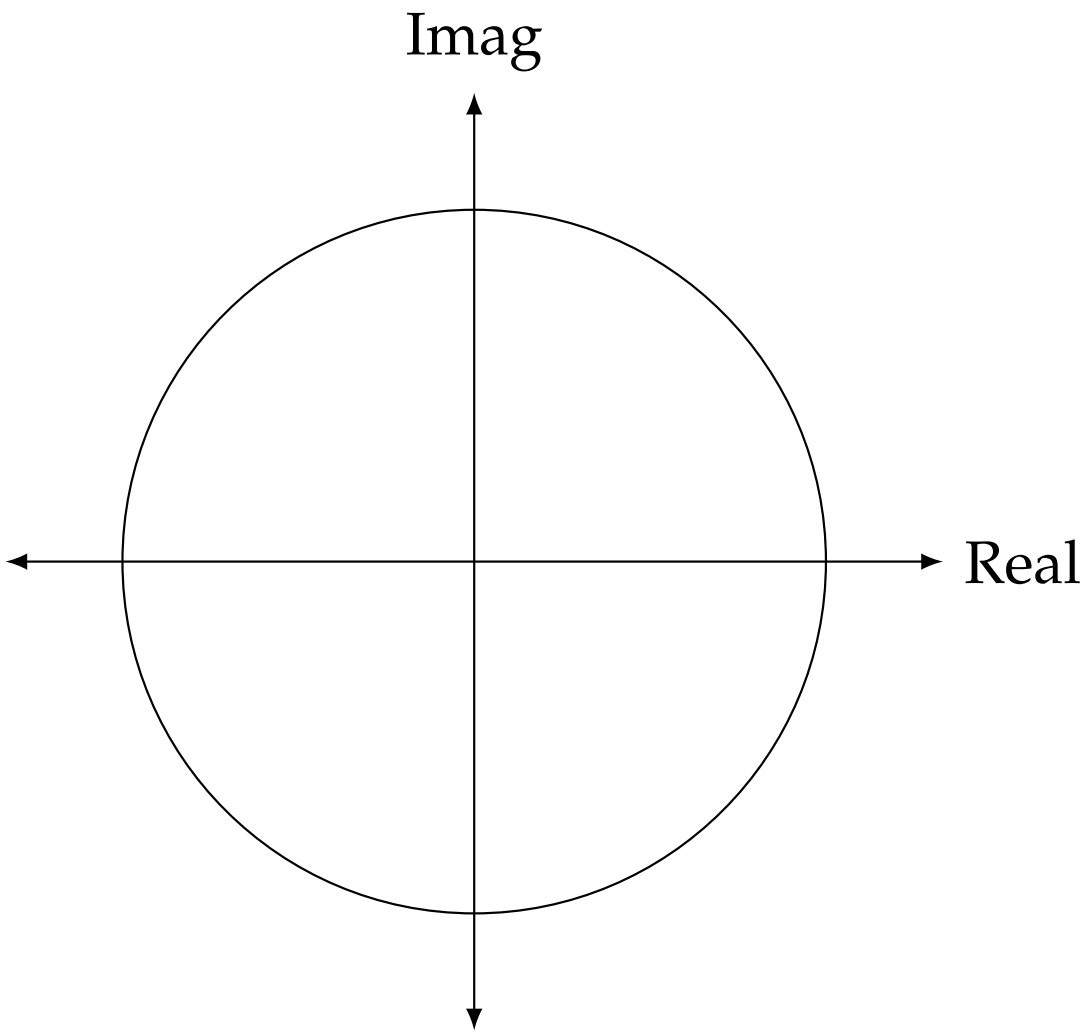
PAM Signals


10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
P_e
0
5
10
15
20
SNR/bit, dB
2-PAM
4-PAM
8-PAM
16-PAM

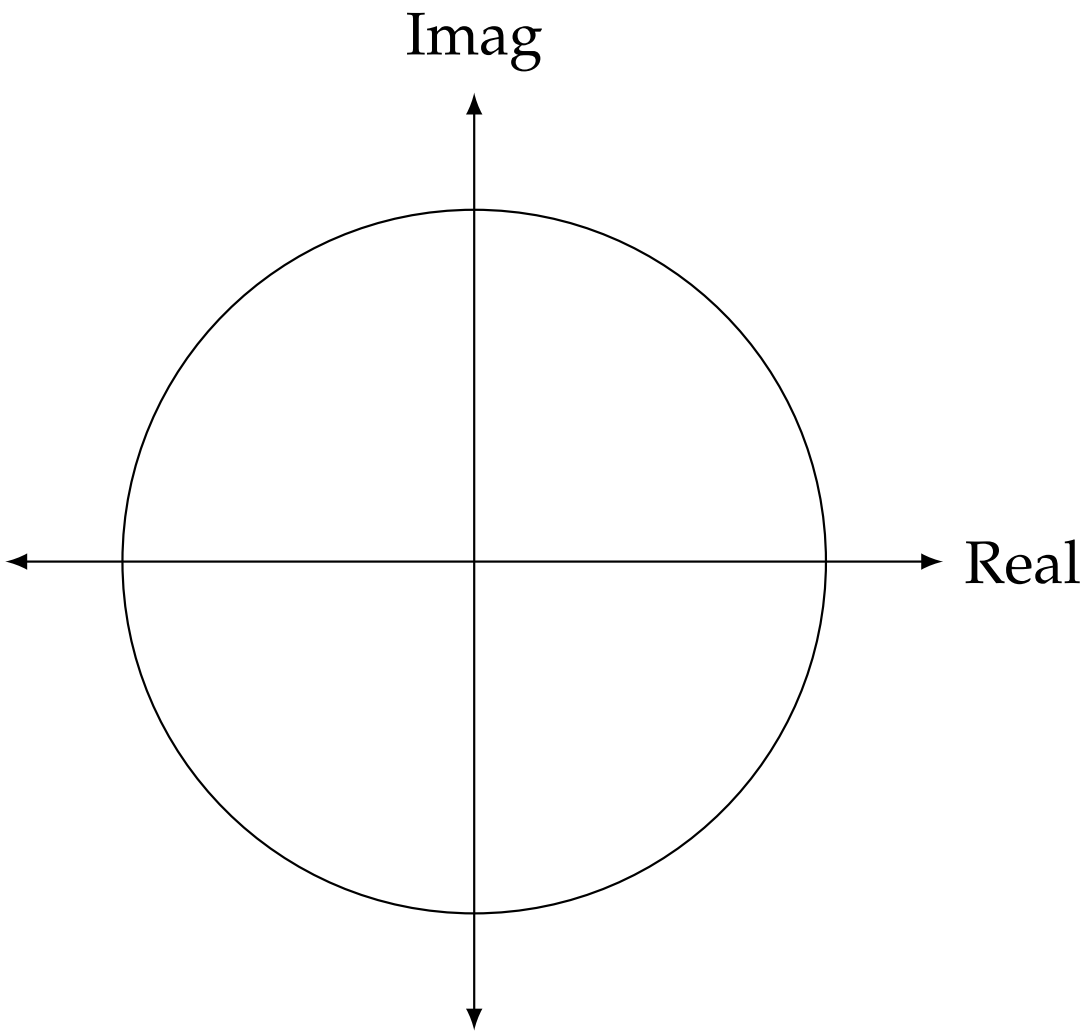
PSK Signals


10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
P_e
−5
0
5
10
15
20
25
SNR/bit, dB
2-PSK
4-PSK
8-PSK
16-PSK
32-PSK

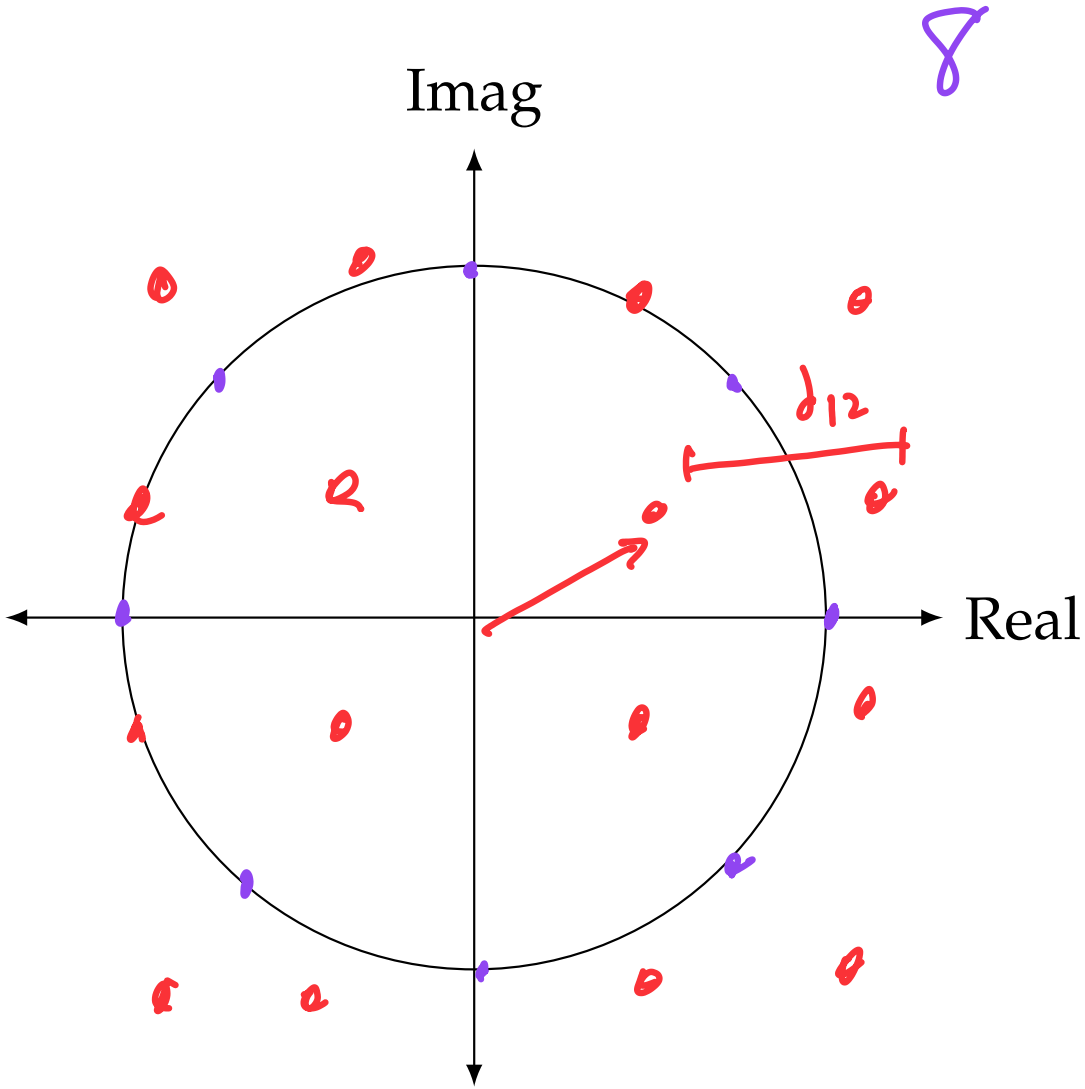
Constellation Diagram: 2-PAM



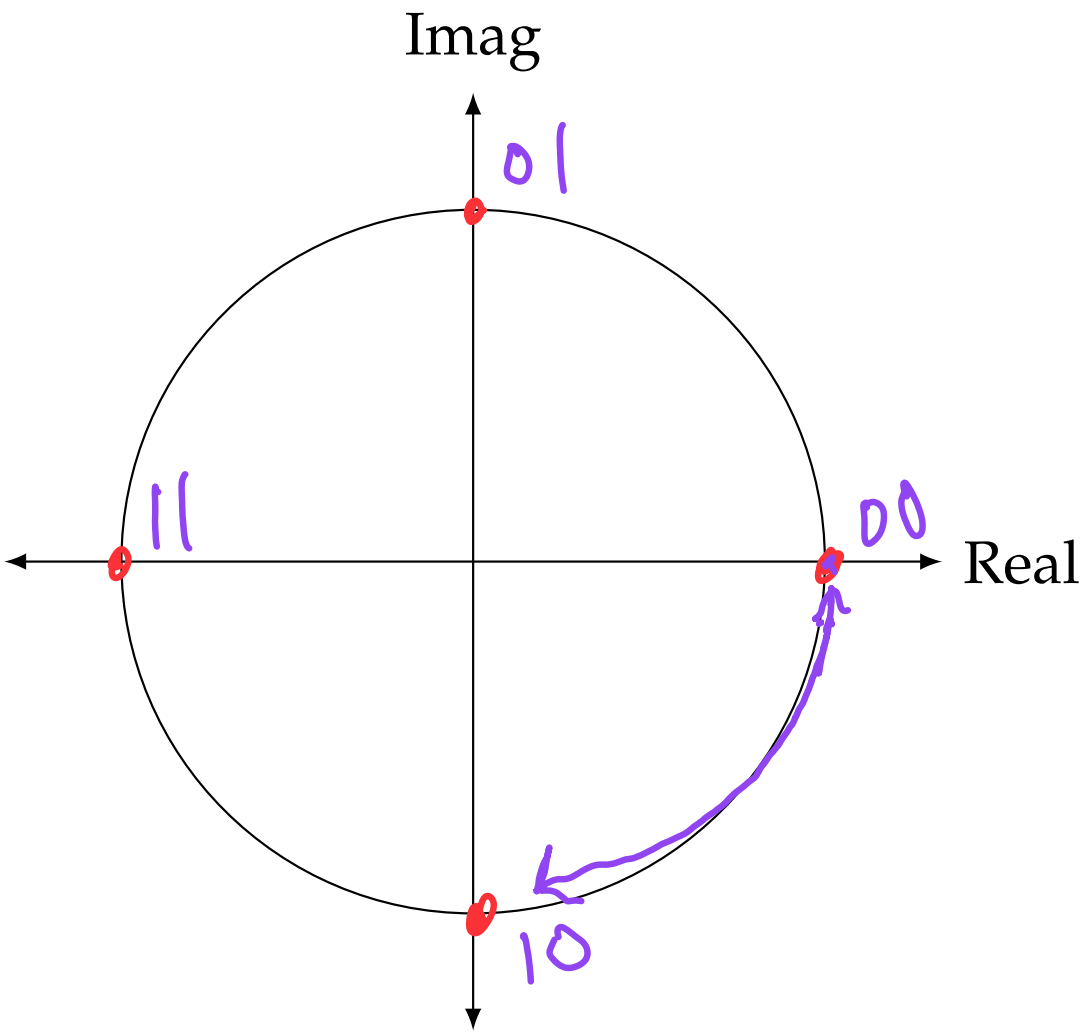
Constellation Diagram: 4 PAM



Constellation Diagram: M-PSK



Gray Coding



Gray Coding

Each successive symbol varies only by 1 bit.

Map out Gray Code for 8-PSK

