Learning Objective 4.3: I can apply radar detection theory (PD, FAR, dwell time) to determine detection performance under noise.

Show your work. A **Documentation** line at the end is required (write **None** if you did not collaborate).

1. **Threshold and false alarms.** A square-law detector tests one range cell that holds only receiver noise of mean power P_n. The power z in the cell is exponentially distributed,

$$p(z) = \frac{1}{P_n}\, e^{-z/P_n}.$$

(a) The detector sets its threshold at $T = 10P_n$. Find the probability of false alarm.

P_{fa} =

(b) Find the threshold, in dB above the mean noise, that gives $P_{fa} = 10^{-5}$.

T =

(c) A classmate looks at a range profile of 200 noise-only cells, sees one cell 7 dB above the mean noise, and concludes that a target must be there. Find the probability that one noise-only cell reaches 7 dB , and explain whether the conclusion is justified.

2. **False-alarm rate.** A radar forms a range-Doppler map of 256 range cells by 32 Doppler cells and refreshes it every 32 ms. The detector tests every cell of every map against the threshold.

   (a) Find the number of threshold decisions per second.

   (b) With $P_{fa} = 10^{-6}$, find the false-alarm rate and the mean time between false alarms.

   (c) The specification allows one false alarm every 10 minutes. Find the required P_{fa} and the threshold in dB above the mean noise.

   P_{fa} =

   T =

   (d) The false-alarm rate fell by a factor of 150 between parts (b) and (c). Explain why the threshold moved by only 1.3 dB .

3. **Required SNR and integration.** A detector must reach $P_d = 0.8$ at $P_{fa} = 10^{-5}$ on a steady target. Albersheim's equation gives the required single-pulse SNR as

$$A = \ln \frac{0.62}{P_{fa}}, \quad B = \ln \frac{P_d}{1 - P_d}, \quad \mathrm{SNR}_{\mathrm{dB}} = 10 \log_{10} \left(A + 0.12AB + 1.7B \right).$$

(a) Find the required SNR in dB.

SNR =

(b) One chirp gives the target an SNR of 2.0 dB . Find the smallest number of chirps that, integrated coherently, meets the requirement.

N =

(c) The Doppler FFT needs a power of two, so the radar uses $N = 16$ chirps, one every 1.0 ms. Find the dwell time and the Doppler resolution.

T_{d} =

Δf_D =

(d) The target is a cart moving at 3.0 m/s and the range resolution is 0.30 m. Show whether the target stays in one range cell during the dwell, and find the longest dwell that keeps it there.

(e) The radar instead adds the powers of 16 chirps (noncoherent integration). Albersheim's equation for N pulses is

$$\begin{aligned}\mathrm{SNR}_{\mathrm{dB}} = &-5\log_{10}(N) \\ &+\left(6.2+\frac{4.54}{\sqrt{N+0.44}}\right)\log_{10}(A+0.12AB+1.7B)\,.\end{aligned}$$

Find the required SNR per chirp, and how much more it is than with coherent integration of the same 16 chirps.

4. **Detection range.** A target gives 18 dB SNR on a single chirp at 50 m. The radar integrates 32 chirps coherently and needs 13.1 dB for $P_d = 0.9$ at $P_{fa} = 10^{-6}$. The received power falls as $1/R^4$ and the noise does not depend on range.

   (a) Find the integrated SNR at 50 m and the margin over the requirement.

   margin = ____

   (b) Find the largest range at which the target is still detected with $P_d = 0.9$.

   R_{max} = ____

   (c) The target is a person whose echo fluctuates from look to look (Swerling case 1), for which the same P_d and P_{fa} need 21.1 dB . Find the new maximum range and state in one sentence why a fluctuating target needs more SNR.

5. **A fixed threshold.** The threshold of a detector sits where it gives $P_{fa} = 10^{-6}$ at mean noise P_n. Later, interference raises the noise by 2 dB and the threshold stays where it was.

   (a) Find the new probability of false alarm, and the factor by which the false-alarm rate grows.

   P'_{fa} = ☐

   factor = ☐

   (b) An operator proposes raising the threshold permanently by 2 dB to cover the interference. Explain what that does to P_d when the interference is absent.

   (c) Describe how the processor can set the threshold so that P_{fa} stays at 10^{-6} whether or not the interference is present.

Documentation: _____