Learning Objective 4.3: I can apply radar detection theory (PD, FAR, dwell time) to determine detection performance under noise.

Show your work. A **Documentation** line at the end is required (write **None** if you did not collaborate).

1. **Threshold and false alarms.** A square-law detector tests one range cell that holds only receiver noise of mean power P_n. The power z in the cell is exponentially distributed,

$$p(z) = \frac{1}{P_n} e^{-z/P_n}.$$

(a) The detector sets its threshold at $T = 10P_n$. Find the probability of false alarm.

P_{fa} is the area of the noise density above the threshold:

$$P_{fa} = \int_T^\infty \frac{1}{P_n} e^{-z/P_n}\, dz = e^{-T/P_n}$$
$$= e^{-10} = 4.5 \times 10^{-5}$$

P_{fa} = 4.5×10^{-5}

(b) Find the threshold, in dB above the mean noise, that gives $P_{fa} = 10^{-5}$.

$$\frac{T}{P_n} = \ln \frac{1}{P_{fa}} = \ln 10^5 = 11.5$$
$$10\log_{10}(11.5) = 10.6 \text{ dB}$$

T = 10.6 dB

(c) A classmate looks at a range profile of 200 noise-only cells, sees one cell 7 dB above the mean noise, and concludes that a target must be there. Find the probability that one noise-only cell reaches 7 dB , and explain whether the conclusion is justified.

7 dB is $10^{0.7} = 5.0$ times P_n, so

$$P(z > 5P_n) = e^{-5.0} = 6.7 \times 10^{-3}.$$

In 200 cells we expect $200 \times 0.0067 = 1.3$ such cells from noise alone. One cell at 7 dB is ordinary noise, not evidence of a target.

2. **False-alarm rate.** A radar forms a range-Doppler map of 256 range cells by 32 Doppler cells and refreshes it every 32 ms. The detector tests every cell of every map against the threshold.

   (a) Find the number of threshold decisions per second.

   $$\frac{256 \times 32}{0.032\ \text{s}} = 2.56 \times 10^5\ \text{s}^{-1}$$

   2.56×10^5/s

   (b) With $P_{fa} = 10^{-6}$, find the false-alarm rate and the mean time between false alarms.

   $$\text{FAR} = P_{fa} \times 2.56 \times 10^5\ \text{s}^{-1} = 0.256\ \text{s}^{-1}$$
   $$1/\text{FAR} = 3.9\ \text{s}$$

   0.26/s, every 3.9 s

   (c) The specification allows one false alarm every 10 minutes. Find the required P_{fa} and the threshold in dB above the mean noise.

   $$P_{fa} = \frac{1/600\ \text{s}^{-1}}{2.56 \times 10^5\ \text{s}^{-1}} = 6.5 \times 10^{-9}$$
   $$\frac{T}{P_n} = \ln \frac{1}{6.5 \times 10^{-9}} = 18.8$$
   $$10\log_{10}(18.8) = 12.8\ \text{dB}$$

   P_{fa} = 6.5×10^{-9}

   T = 12.8 dB

   (d) The false-alarm rate fell by a factor of 150 between parts (b) and (c). Explain why the threshold moved by only 1.3 dB .

   The threshold depends on the logarithm of $1/P_{fa}$: T/P_n went from $\ln 10^6 = 13.8$ to 18.8, a ratio of 1.36, or 1.3 dB . The exponential noise tail falls so fast that a small move in the threshold removes most of the crossings.

3. **Required SNR and integration.** A detector must reach $P_d = 0.8$ at $P_{fa} = 10^{-5}$ on a steady target. Albersheim's equation gives the required single-pulse SNR as

$$A = \ln \frac{0.62}{P_{fa}}, \quad B = \ln \frac{P_d}{1 - P_d}, \quad \text{SNR}_{\text{dB}} = 10\log_{10}\left(A + 0.12AB + 1.7B\right).$$

(a) Find the required SNR in dB.

$$A = \ln\left(6.2 \times 10^4\right) = 11.03, \qquad B = \ln 4 = 1.386$$
$$\text{SNR}_{\text{dB}} = 10\log_{10}\left(11.03 + 1.84 + 2.36\right)$$
$$= 10\log_{10}\left(15.2\right) = 11.8 \text{ dB}$$

SNR = 11.8 dB

(b) One chirp gives the target an SNR of 2.0 dB. Find the smallest number of chirps that, integrated coherently, meets the requirement.

Coherent integration multiplies the SNR by N, so it must supply $11.8 - 2.0 = 9.8$ dB :

$$N \geqslant 10^{9.8/10} = 9.6 \quad \Rightarrow \quad N = 10$$

N = 10

(c) The Doppler FFT needs a power of two, so the radar uses $N = 16$ chirps, one every 1.0 ms. Find the dwell time and the Doppler resolution.

$$T_{\text{d}} = NT_c = 16 \times 1.0 \text{ ms} = 16 \text{ ms}$$
$$\Delta f_D = \frac{1}{T_{\text{d}}} = 62.5 \text{ Hz}$$

T_{d} = 16 ms

Δf_D = 62.5 Hz

(d) The target is a cart moving at 3.0 m/s and the range resolution is 0.30 m. Show whether the target stays in one range cell during the dwell, and find the longest dwell that keeps it there.

During the dwell the cart moves 3.0 m/s $\times$ 0.016 s $=$ 0.048 m, well under one 0.30 m cell. The longest dwell is

$$\frac{\Delta R}{v} = \frac{0.30 \text{ m}}{3.0 \text{ m/s}} = 0.10 \text{ s},$$

about 100 chirps.

(e) The radar instead adds the powers of 16 chirps (noncoherent integration). Albersheim's equation for N pulses is

$$\begin{aligned}\mathrm{SNR}_{\mathrm{dB}} = &-5\log_{10}(N) \\ &+\left(6.2+\frac{4.54}{\sqrt{N+0.44}}\right)\log_{10}(A+0.12AB+1.7B)\,.\end{aligned}$$

Find the required SNR per chirp, and how much more it is than with coherent integration of the same 16 chirps.

$$\begin{aligned}\mathrm{SNR}_{\mathrm{dB}} &= -5\log_{10}(16) \\ &\quad +\left(6.2+\frac{4.54}{4.05}\right)\log_{10}(15.2) \\ &= -6.02+7.32\times 1.182 = 2.64\,\mathrm{dB}\end{aligned}$$

Coherent integration of 16 chirps needs $11.83-12.04=-0.21$ dB per chirp, so the noncoherent detector needs 2.9 dB more.

4. **Detection range.** A target gives 18 dB SNR on a single chirp at 50 m. The radar integrates 32 chirps coherently and needs 13.1 dB for $P_d = 0.9$ at $P_{fa} = 10^{-6}$. The received power falls as $1/R^4$ and the noise does not depend on range.

   (a) Find the integrated SNR at 50 m and the margin over the requirement.

   margin = 20.0 dB

   $\text{SNR}_{32} = 18 \text{ dB} + 10\log_{10}(32) = 18 + 15.05 = 33.1 \text{ dB}$

   $\text{margin} = 33.1 - 13.1 = 20.0 \text{ dB}$

   (b) Find the largest range at which the target is still detected with $P_d = 0.9$.

   $R_{\max}$ = 158 m

   The SNR falls 40 dB per decade of range, so the margin sets the range ratio:

   $$R_{\max} = 50 \text{ m} \times 10^{20.0/40}$$
   $$= 50 \text{ m} \times 3.16 = 158 \text{ m}$$

   (c) The target is a person whose echo fluctuates from look to look (Swerling case 1), for which the same P_d and P_{fa} need 21.1 dB . Find the new maximum range and state in one sentence why a fluctuating target needs more SNR.

   The margin drops to $33.1 - 21.1 = 12.0$ dB , so

   $$R_{\max} = 50 \text{ m} \times 10^{12.0/40} = 100 \text{ m}.$$

   A fluctuating echo sometimes fades well below its average, and on those looks it misses the threshold, so the average SNR must be higher to keep P_d at 0.9.

5. **A fixed threshold.** The threshold of a detector sits where it gives $P_{fa} = 10^{-6}$ at mean noise P_n. Later, interference raises the noise by 2 dB and the threshold stays where it was.

   (a) Find the new probability of false alarm, and the factor by which the false-alarm rate grows.

   The noise rises by $g = 10^{0.2} = 1.585$, so

   $$P'_{fa} = e^{-T/(gP_n)} = P_{fa}^{1/g}$$
   $$= \left(10^{-6}\right)^{0.631} = 1.6 \times 10^{-4}$$

   The false-alarm rate grows by a factor of 160.

   P'_{fa} = 1.6×10^{-4}

   factor = 160

   (b) An operator proposes raising the threshold permanently by 2 dB to cover the interference. Explain what that does to P_d when the interference is absent.

   With normal noise the raised threshold sits $11.4 + 2 = 13.4$ dB above the mean noise, which is $P_{fa} = e^{-21.9} \approx 3 \times 10^{-10}$. False alarms are rare, but a target that just met $P_d = 0.9$ at the old threshold now falls short of the new one much more often, so P_d drops. A fixed threshold can protect against false alarms or missed targets, not both.

   (c) Describe how the processor can set the threshold so that P_{fa} stays at 10^{-6} whether or not the interference is present.

   Estimate the noise power from the cells around the cell under test, skipping a few guard cells next to it, and set the threshold as a fixed multiple of that local average. The threshold then rises and falls with the noise, which keeps P_{fa} constant. That is CFAR detection, the subject of Lesson 30.

Documentation: _____