Learning Objective 4.2: I can calculate range resolution, unambiguous range, and Doppler shift for a given radar waveform.

Show your work. Use $c = 3.00 \times 10^8$ m/s. A **Documentation** line at the end is required (write **None** if you did not collaborate).

1. **Range and unambiguous range.** A pulsed surveillance radar transmits a pulse every 0.5 ms, a pulse repetition frequency of 2 kHz.

   (a) An echo arrives 40 μs after its pulse. Find the target's range.

   The wave covers the range twice:

   $$R = \frac{c\tau}{2} = \frac{(3.00 \times 10^8)\,(40 \times 10^{-6})}{2} = 6.00 \text{ km}$$

   R = **6.00 km**

   (b) Find the unambiguous range.

   Every echo must return before the next pulse leaves:

   $$R_{\text{ua}} = \frac{c}{2\,\text{PRF}} = \frac{3.00 \times 10^8}{2\,(2000)} = 75 \text{ km}$$

   R_{ua} = **75 km**

   (c) A large aircraft is 90 km away. At what range does the radar report it?

   Its echo arrives $2(90 \times 10^3)/c = 600$ μs after its pulse, which is 100 μs after the next pulse. The receiver times it from that pulse:

   $$R_{\text{apparent}} = 90 - 75 = 15 \text{ km}$$

   R_{apparent} = **15 km**

   (d) The radar must report every target out to 120 km without ambiguity. Find the highest PRF it can use.

   $$\text{PRF}_{\text{max}} = \frac{c}{2R_{\text{ua}}} = \frac{3.00 \times 10^8}{2\,(120 \times 10^3)} = 1.25 \text{ kHz}$$

   PRF = **1.25 kHz**

   (e) The operator switches the PRF from 2 kHz to 1.25 kHz and the aircraft of part (c) jumps from 15 km to 90 km. In one or two sentences, explain how this change shows which reading is true.

   A target inside the unambiguous range of both PRFs reports the same range at both. The 15 km reading moved when the PRF changed, so it was folded; at 1.25 kHz the unambiguous range is 120 km, and the 90 km reading is the true one.

2. **Range resolution.** A radar resolves two targets when their echoes do not overlap in time.

(a) A radar transmits a $0.5\ \mu$s pulse. Find its range resolution.

$$\Delta R = \frac{c\,\tau_p}{2} = \frac{(3.00\times10^{8})\,(0.5\times10^{-6})}{2} = 75\text{ m}$$

$\Delta R =$ 75 m

(b) Two aircraft fly in formation, 50 m apart in range. Describe what the radar of part (a) reports, and why.

Their echoes start $2(50)/c = 0.33\ \mu$s apart, less than the $0.5\ \mu$s pulse, so they overlap. The radar reports one target, with an echo longer than a single aircraft's and an amplitude set by the two echoes' relative phase.

(c) Find the bandwidth a waveform needs to resolve targets 0.30 m apart.

$$B = \frac{c}{2\Delta R} = \frac{3.00\times10^{8}}{2\,(0.30)} = 500\text{ MHz}$$

$B =$ 500 MHz

(d) ADI's FMCW lab halves the PHASER's chirp bandwidth to 250 MHz. Find the new range resolution.

$$\Delta R = \frac{c}{2B} = \frac{3.00\times10^{8}}{2\,(250\times10^{6})} = 0.60\text{ m}$$

$\Delta R =$ 0.60 m

Halving the bandwidth doubles the resolution cell.

(e) Two corner reflectors sit at 1.50 m and 1.70 m from the kit. State whether the 500 MHz chirp separates them, and say what you would change to separate them.

They are 0.20 m apart, inside the 0.30 m resolution of a 500 MHz chirp, so they appear as one target. More bandwidth would separate them (750 MHz gives 0.20 m, just at the limit), or you can move one reflector so that they are more than 0.30 m apart in range.

(f) A classmate proposes doubling the transmit power to improve the range resolution. Explain in one or two sentences why that does not work.

Resolution depends only on the waveform's bandwidth, $\Delta R = c/2B$. More power makes both echoes stronger but does not shorten them, so they still overlap.

3. **The Doppler shift.** The Doppler shift of an echo is $f_D = 2v_r/\lambda$, positive for a target closing on the radar.

(a) ADI's CW radar lab quotes 716 Hz at 2.4 GHz and 2982 Hz at 10 GHz for a car at 100 mph (1 mph $= 0.44704$ m/s). Recompute both.

$v = 44.7$ m/s; $\lambda = 0.125$ m at 2.4 GHz and 0.030 m at 10 GHz:

$$f_D = \frac{2\,(44.7)}{0.125} = 715 \text{ Hz}$$

$$f_D = \frac{2\,(44.7)}{0.030} = 2980 \text{ Hz}$$

ADI's slightly larger values come from $c = 2.998 \times 10^8$ m/s.

f_D = 715 Hz

f_D = 2980 Hz

(b) A target moves directly away from a 10 GHz radar at 15 m/s. Find its Doppler shift, with its sign.

The range is opening, so $v_r = -15$ m/s:

$$f_D = \frac{2\,(-15)}{0.030} = -1000 \text{ Hz}$$

f_D = −1000 Hz

(c) An aircraft flies at 200 m/s on a heading 60° off the line of sight, toward the radar. Find the Doppler shift at 10 GHz.

Only the radial component counts:

$$v_r = 200 \cos 60^\circ = 100 \text{ m/s}$$

$$f_D = \frac{2\,(100)}{0.030} = 6.67 \text{ kHz}$$

f_D = 6.67 kHz

(d) The same aircraft turns to fly across the line of sight. State its Doppler shift and what that means for a radar that separates targets by Doppler.

At $\alpha = 90^\circ$, $v_r = 0$ and $f_D = 0$. The aircraft has the same Doppler as the ground, so a radar that rejects zero-Doppler returns loses it until it turns again.

(e) Explain in two or three sentences why an approaching target raises the echo's frequency.

Each wave crest reflects from the target when the target is a little closer than it was for the crest before, so each crest has a shorter round trip. The crests arrive closer together in time than they were sent, which is a higher frequency. Equivalently, the echo phase $-4\pi R/\lambda$ increases as R shrinks, and a steadily increasing phase is a positive frequency offset.

4. **Doppler resolution and ambiguity.** ADI's range-Doppler lab sends a burst of 256 chirps, 0.5 ms apart, at about 10 GHz ($\lambda = 3.0$ cm). Each chirp gives one sample of the target's Doppler phase.

   (a) Find the burst length, its frequency resolution, and its velocity resolution.

$$T = 256\,(0.5\text{ ms}) = 128\text{ ms}$$
$$\Delta f = \frac{1}{T} = 7.8\text{ Hz}$$
$$\Delta v = \frac{\lambda}{2T} = \frac{0.030}{2\,(0.128)} = 0.12\text{ m/s}$$

   T = 128 ms

   Δv = 0.12 m/s

   (b) Find the unambiguous velocity.

   The chirps sample the Doppler at the PRF, 2 kHz, so shifts within ± 1 kHz are unambiguous:

$$v_{\text{ua}} = \pm\frac{\lambda}{4\,T_{\text{PRI}}} = \pm\frac{0.030}{4\,(0.5\times 10^{-3})} = \pm 15\text{ m/s}$$

   v_{ua} = ± 15 m/s

   (c) ADI suggests changing the number of chirps to 128. State what happens to the velocity resolution and to the unambiguous velocity.

   The burst halves to 64 ms, so Δv doubles to 0.23 m/s. The chirps are still 0.5 ms apart, so v_{ua} stays at ± 15 m/s.

   (d) A car closes on this radar at 25 m/s. Find its true Doppler shift and the velocity the radar reports.

$$f_D = \frac{2\,(25)}{0.030} = 1667\text{ Hz}$$

   That is outside ± 1000 Hz, so it aliases by one PRF:

$$f_{\text{rep}} = 1667 - 2000 = -333\text{ Hz}$$
$$v_{\text{rep}} = \frac{\lambda f_{\text{rep}}}{2} = -5\text{ m/s}$$

   The radar reports a target moving away at 5 m/s.

   f_D = 1667 Hz

   v_{rep} = -5 m/s

   (e) Find the unambiguous range of this pulse spacing, and show that the product $R_{\text{ua}}v_{\text{ua}}$ equals $c\lambda/8$.

$$R_{\text{ua}} = \frac{c\,T_{\text{PRI}}}{2} = 75\text{ km}$$
$$R_{\text{ua}}v_{\text{ua}} = \left(75\times 10^{3}\right)(15) = 1.125\times 10^{6}\text{ m}^2/\text{s}$$
$$\frac{c\lambda}{8} = \frac{\left(3.00\times 10^{8}\right)(0.030)}{8} = 1.125\times 10^{6}\text{ m}^2/\text{s}$$

   R_{ua} = 75 km

5. **CW radar, the fan, and micro-Doppler.** ADI's CW lab points an unmodulated 10 GHz radar at a fan. Its script forms each spectrum from 65,536 samples taken at 0.6 MHz.

   (a) Find the time each spectrum spans, its bin width, and the velocity one bin represents.

$$T = \frac{65{,}536}{0.6 \times 10^6} = 109 \text{ ms}$$
$$\Delta f = \frac{1}{T} = 9.2 \text{ Hz}$$
$$\Delta v = \frac{\lambda \, \Delta f}{2} = \frac{(0.030)\,(9.2)}{2} = 0.14 \text{ m/s}$$

   Δf = 9.2 Hz

   Δv = 0.14 m/s

   (b) The fan is 0.30 m across and turns at 1200 rpm, seen edge-on. Find the blade-tip speed and the edges of the band its blades fill.

$$v_{\text{tip}} = \pi D \, \frac{\text{rpm}}{60} = \pi\,(0.30)\,(20) = 18.8 \text{ m/s}$$
$$f_{\text{edge}} = \pm \frac{2\,(18.8)}{0.030} = \pm 1257 \text{ Hz}$$

   v_{tip} = 18.8 m/s

   f_{edge} = ±1257 Hz

   (c) A different fan, 0.25 m across, shows a band with edges at ± 800 Hz. Find its speed in rpm.

$$v_{\text{tip}} = \frac{\lambda f_{\text{edge}}}{2} = \frac{(0.030)\,(800)}{2} = 12.0 \text{ m/s}$$
$$\text{rpm} = \frac{60\, v_{\text{tip}}}{\pi D} = \frac{60\,(12.0)}{\pi\,(0.25)} = 917$$

   rpm = 917

   (d) Explain in one or two sentences why this CW radar measures the fan's blade speed but not its range.

   An unmodulated tone has no feature to time: every cycle of the echo looks like every other, so there is no delay to measure. The frequency shift is still measurable, so the CW radar gives speed only.

   (e) Place the fan of part (b) in front of the range-Doppler radar of Question 4. Explain what happens to its blade returns there, and why the CW radar avoids it.

   The tips reach ± 18.8 m/s, beyond that radar's ± 15 m/s, so the outer parts of the blades alias and wrap to the opposite side of the Doppler axis. The CW radar samples continuously at 0.6 MHz, so its unambiguous window is hundreds of kilohertz wide and the whole ± 1.26 kHz band fits.

(f) A person walks toward the radar at 1.3 m/s, and the swinging limbs reach 4.0 m/s. Find the Doppler of the torso and the peak Doppler of the limbs.

f_{torso} = 87 Hz

f_{limb} = 267 Hz

$$f_{\text{torso}} = \frac{2\,(1.3)}{0.030} = 87 \text{ Hz}$$

$$f_{\text{limb}} = \frac{2\,(4.0)}{0.030} = 267 \text{ Hz}$$

The limbs' micro-Doppler swings about the torso line once per stride.