Learning Objective 4.1: I can apply the radar equation to calculate received power for a given geometry, and account for path loss and radar cross section (RCS).

Show your work. A **Documentation** line at the end is required (write **None** if you did not collaborate).

1. **Friis applied twice.** A radar operates at 10 GHz, so $\lambda = 3.0$ cm. Its target sits at $R = 3$ m.

   (a) Compute the one-way free-space path loss $(4\pi R/\lambda)^2$ in dB.

   $$L_{\text{fs}} = 20\log_{10}\left(\frac{4\pi R}{\lambda}\right) = 20\log_{10}\left(\frac{4\pi(3)}{0.030}\right)$$
   $$= 20\log_{10}(1257) = 62.0 \text{ dB}$$

   L_{fs} = **62.0 dB**

   (b) A one-way link loses power as $1/R^2$, but a radar echo falls as $1/R^4$. Explain where the extra $1/R^2$ comes from, in one or two sentences.

   The transmitted power spreads over a sphere of area $4\pi R^2$ on the way to the target. The target re-radiates what it intercepts, and that echo spreads over a second sphere of area $4\pi R^2$ on the way back, so the two trips each contribute a $1/R^2$.

   (c) The target moves from 3 m to 6 m. By how many dB does the echo power change? By how many dB would the received power on a one-way link change?

   $$\Delta P_{\text{radar}} = -40\log_{10}\left(\frac{6}{3}\right) = -12.0 \text{ dB}$$
   $$\Delta P_{\text{link}} = -20\log_{10}\left(\frac{6}{3}\right) = -6.0 \text{ dB}$$

   radar: **−12.0 dB**

   link: **−6.0 dB**

   (d) The radar equation in dB carries the constant $10\log_{10}\left((4\pi)^3\right)$. Evaluate it.

   $$10\log_{10}\left((4\pi)^3\right) = 30\log_{10}(12.57) = 33.0 \text{ dB}$$

   constant = **33.0 dB**

2. **Three shapes.** At 10 GHz ($\lambda = 3.0$ cm), compare three targets of the same size: a sphere 20 cm in diameter, a square flat plate 20 cm on a side facing the radar, and a trihedral corner reflector with 20 cm edges, viewed along its symmetry axis. Give each peak RCS in m^2 and in dBsm.

   (a) Find the sphere's peak RCS.

   The circumference is $2\pi(0.1)/0.030 = 21$ wavelengths, so the sphere is in the optical region and returns its shadow area:

   $$\sigma = \pi a^2 = \pi(0.1)^2 = 0.0314 \text{ m}^2 = -15.0 \text{ dBsm}$$

   σ = -15.0 dBsm

   (b) Find the flat plate's peak RCS.

   The plate intercepts over A and re-radiates with the uniform-aperture gain $4\pi A/\lambda^2$:

   $$\sigma = \frac{4\pi A^2}{\lambda^2} = \frac{4\pi(0.04)^2}{(0.030)^2}$$
   $$= 22.3 \text{ m}^2 = +13.5 \text{ dBsm}$$

   σ = $+13.5$ dBsm

   (c) Find the corner reflector's peak RCS.

   $$\sigma = \frac{4\pi a^4}{3\lambda^2} = \frac{4\pi(0.2)^4}{3\,(0.030)^2}$$
   $$= 7.45 \text{ m}^2 = +8.7 \text{ dBsm}$$

   σ = $+8.7$ dBsm

   (d) The plate has the largest peak RCS, yet radar ranges calibrate against corner reflectors. Explain why, and state how the corner's peak RCS compares with the plate's.

   The plate returns its echo in a beam as narrow as its own aperture pattern, so its RCS falls 3 dB within a few degrees of normal and a small misalignment changes the calibration by many dB. The corner's three perpendicular faces send every ray back the way it came, so its RCS stays within 3 dB of the peak over about $\pm 19^\circ$. Its peak RCS is about 4.8 dB below that of the plate of the same size, because only the hexagonal part of its opening returns a triple bounce.

3. **Noise and G/T.** A receiver at room temperature has an antenna that sees $T_A = 290$ K. At $T_0 = 290$ K the noise density is $kT_0 = -174$ dBm/Hz.

   (a) Find the thermal noise power kT_0B in a 2 kHz bandwidth.

$$N = -174 + 10\log_{10}(2000)$$
$$= -174 + 33.0 = -141.0 \text{ dBm}$$

N = -141.0 dBm

   (b) The receiver has a 3 dB noise figure. Find its equivalent noise temperature and the system noise temperature.

$$T_e = T_0(F-1) = 290\left(10^{0.3}-1\right) = 289 \text{ K}$$
$$T_{\text{sys}} = T_A + T_e = 290 + 289 = 579 \text{ K}$$

T_e = 289 K

T_{sys} = 579 K

   (c) The receive antenna has 20 dBi of gain. Find G/T in dB/K.

$$G/T = 20 - 10\log_{10}(579)$$
$$= 20 - 27.6 = -7.6 \text{ dB/K}$$

G/T = -7.6 dB/K

   (d) Every PHASER element feeds a low-noise amplifier with 24 dB of gain before anything else. Explain why the first amplifier sets the noise figure of the whole receiver.

   The first amplifier adds its own noise and then multiplies both the signal and that noise by its gain. Noise added by later stages is small next to the amplified noise from the first stage, so their contribution, referred to the input, is divided by the first stage's gain of 24 dB . The receiver's noise figure is therefore close to the first amplifier's 1.3 dB .

4. **The PHASER link budget.** The PHASER transmits $P_t = -16$ dBm at 10 GHz through a 6 dBi antenna and receives on its array with $G_r = 15$ dBi . The target is a trihedral corner reflector with 10 cm edges ($\sigma = -3.3$ dBsm). The receiver noise figure is 2 dB , the antenna sees 290 K, and the bandwidth is 1 MHz.

   (a) Find the echo power at $R = 5$ m.

$$\begin{aligned} P_r &= P_t + G_t + G_r + 10\log_{10}(\lambda^2) \\ &\quad + \sigma - 33.0 - 40\log_{10}(R) \\ &= -16.00 + 6.00 + 15.00 - 30.46 \\ &\quad - 3.32 - 32.98 - 27.96 \\ &= -89.7 \text{ dBm} \end{aligned}$$

P_r = -89.7 dBm

   (b) Find the SNR at 5 m.

With $T_A = T_0$ the noise is kT_0BF:

$$N = -174 + 60 + 2 = -112.0 \text{ dBm}$$
$$\text{SNR} = -89.7 - (-112.0) = 22.3 \text{ dB}$$

SNR = 22.3 dB

   (c) The detector needs an SNR of at least 13 dB . Find the maximum range.

The echo has $22.3 - 13 = 9.3$ dB to spare at 5 m, and range enters at 40 dB per decade:

$$R_{\max} = 5 \text{ m} \times 10^{9.3/40} = 8.5 \text{ m}$$

$R_{\max}$ = 8.5 m

   (d) What transmit power would extend the maximum range to 25 m, with everything else unchanged?

$$\Delta P_t = 40\log_{10}\left(\frac{25}{8.5}\right) = 18.7 \text{ dB}$$
$$P_t = -16 + 18.7 = +2.7 \text{ dBm}$$

Twice the range takes about 16 times the power.

P_t = $+2.7$ dBm

5. **Seeing a drone.** The same PHASER setup now looks for a small drone with $\sigma = -20$ dBsm , still in a 1 MHz bandwidth and with a 13 dB threshold.

   (a) Find the maximum range against the drone.

   RCS enters at one quarter of its dB value. The drone is 16.68 dB below the corner, so

   $$R_{\text{max}} = 8.53\ \text{m} \times 10^{-16.68/40} = 3.27\ \text{m}$$

   R_{max} = 3.27 m

   (b) How many dB must the budget improve to detect the drone at 10 m?

   $$\Delta = 40\log_{10}\left(\frac{10}{3.27}\right) = 19.4\ \text{dB}$$

   Δ = 19.4 dB

   (c) Find the receiver bandwidth that provides the whole improvement on its own.

   Noise power is proportional to bandwidth, so

   $$B = \frac{1\ \text{MHz}}{10^{19.4/10}} = 11.5\ \text{kHz}$$

   B = 11.5 kHz

   (d) The same 19.4 dB could come from about 87 times the transmit power instead. State which change you would make on the PHASER and why, in two or three sentences.

   Narrow the bandwidth. A narrower filter, or a longer FFT on the processed signal, removes noise without any new hardware, while 87 times the power needs a larger amplifier, more prime power, and more heat removal. The narrower bandwidth requires a longer observation of the target, which a radar obtains by integrating over more time, and Lesson 28 shows how the FMCW processing does exactly that.

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