Learning Objective 3.9: I can calculate null-steering weights and implement pattern nulls on the ADALM-PHASER.

Show your work. A **Documentation** line at the end is required (write **None** if you did not collaborate).

1. **A null versus more power.** The PHASER views a target on boresight while an interferer sits at $+22.5^\circ$. At the aperture the interferer is 10 dB stronger than the target. The array is uniformly weighted and steered broadside, so its pattern level at $+22.5^\circ$ is the first sidelobe level, -13 dB relative to the peak.

   (a) Compute the signal-to-interference ratio at the beamformer output.

   SIR = ____

   (b) Null steering replaces the sidelobe at $+22.5^\circ$ with a notch measured at -46 dB relative to the new main-lobe peak. Recompute the SIR.

   SIR = ____

   (c) The same 33 dB of margin could also come from raising the transmit power. State what that requires on a radar, in one or two sentences, and say which fix you would specify.

   (d) The interferer is at $+22.5^\circ$ and the target is on boresight. Explain why the array does not reject the interferer simply because it is off axis.

2. **Steering vectors.** The PHASER array has $N = 8$ elements at $d = 14$ mm, receiving the HB100 tone at 10.525 GHz so that $d/\lambda = 0.491$ and $kd = 3.085$ rad. The steering vector for arrival angle θ has entries $v_n(\theta) = e^{jnkd\sin(\theta)}$, and the beamformer response is $y(\theta) = \sum_n w_n v_n(\theta)$.

   (a) In one or two sentences, state what $\vec{v}(\theta)$ describes and what it does *not* depend on.

   (b) Compute the element-to-element phase for a commanded beam at $\theta_0 = 30^\circ$.

   $\Delta\phi$ = ____

   (c) Write the weight w_3 (the fourth element, $n = 3$) that steers the beam to 30°, as a magnitude and an angle.

   w_3 = ____

   (d) With those weights loaded, evaluate $y(30^\circ)$ and express the result in dB relative to one element.

   y = ____

3. **Weight subtraction by hand.** A four-element array with $d = \lambda/2$ (so $kd = 180^\circ$) is steered broadside, giving $\vec{w}_\mathsf{d} = [1, 1, 1, 1]$. An interferer appears at $\theta_1 = 45^\circ$. Use the weight-subtraction rule

$$\vec{w} = \vec{w}_\mathsf{d} - r_\mathsf{n}\vec{w}_\mathsf{n}, \qquad r_\mathsf{n} = \frac{\vec{w}_\mathsf{n}^H \vec{w}_\mathsf{d}}{\vec{w}_\mathsf{n}^H \vec{w}_\mathsf{n}}.$$

(a) Compute the element-to-element phase $\psi_1 = kd\sin(\theta_1)$ at the interferer angle.

ψ_1 =

(b) Evaluate r_n. For a broadside beam it reduces to the normalized uniform array factor at θ_1,

$$r_\mathsf{n} = e^{j(N-1)\psi_1/2}\,\frac{\sin(N\psi_1/2)}{N\sin(\psi_1/2)}.$$

r_n =

(c) Compute the four weights $w_n = 1 - r_\mathsf{n} e^{-jn\psi_1}$ and convert them to PHASER-style settings: element gains as percentages of the largest, and phases in degrees.

(d) Find the loss in the look direction after the weights are rescaled so the largest element sits at 100% gain.

loss =

4. **Main-lobe loss.** Return to the eight-element PHASER array ($kd = 3.085$ rad, broadside beam). Before the weights are rescaled, the response in the look direction falls from N to $N\left(1 - |r_{\mathrm{n}}|^2\right)$.

   (a) An interferer sits at $\theta_1 = 10^\circ$, where the uniform pattern level is -8.0 dB . Compute the loss in the look direction before rescaling.

   loss = ____

   (b) A second interferer sits at $\theta_1 = 30^\circ$, where the uniform pattern level is -33.9 dB . Compute the loss and comment on the result in one sentence.

   loss = ____

   (c) The PHASER's half-power beamwidth is 13.1°. A student asks the array to null an interferer at $\theta_1 = 3^\circ$, where $|r_{\mathrm{n}}| = 0.93$. Explain what the resulting pattern looks like and what the student should do instead.

   (d) Explain, in one or two sentences, why $|r_{\mathrm{n}}|$ is worth computing before calculating any weights.

5. **How deep can the null be?** Each ADAR1000 phase setting lands on a 2.8125° grid and each gain on a 1% grid, so each weight carries an independent error. The errors add in RMS at the null angle, giving a residual response

$$\frac{|y(\theta_1)|}{N} \approx \frac{\epsilon_{\text{rms}}}{\sqrt{N}}, \qquad \epsilon_{\text{rms}} = \sqrt{\sigma_\phi^2 + \sigma_a^2}, \qquad \sigma_\phi = \frac{\text{LSB}}{\sqrt{12}},$$

with σ_ϕ in radians.

(a) Compute the quantization-limited null depth for the eight-element PHASER with its 7-bit phase shifters.

depth = ____

(b) Repeat for a 3-bit phase shifter, ignoring the gain error. Compare your answer with the $-6B$ rule of thumb from L23.

depth = ____

(c) On a course kit, the lab sweep measured the notch 46 to 50 dB below the uniform-taper peak, with the null-steered main lobe 2 dB below that same reference. Compare the measurement with your answer to (a) and state what limits the depth.

(d) State the maximum number of independent nulls an eight-element array can place and explain the counting in one sentence.

Documentation: _____