Learning Objective 3.9: I can calculate null-steering weights and implement pattern nulls on the ADALM-PHASER.

Show your work. A **Documentation** line at the end is required (write **None** if you did not collaborate).

1. **A null versus more power.** The PHASER views a target on boresight while an interferer sits at $+22.5^\circ$. At the aperture the interferer is 10 dB stronger than the target. The array is uniformly weighted and steered broadside, so its pattern level at $+22.5^\circ$ is the first sidelobe level, -13 dB relative to the peak.

   (a) Compute the signal-to-interference ratio at the beamformer output.

   The target sits at the pattern peak and the sidelobe level attenuates the interferer, so both are referred to the same peak:

   $$\begin{aligned}\text{SIR} &= 0 \text{ dB} - (10 \text{ dB} - 13 \text{ dB}) \\ &= 3 \text{ dB}\end{aligned}$$

   SIR = 3 dB

   (b) Null steering replaces the sidelobe at $+22.5^\circ$ with a notch measured at -46 dB relative to the new main-lobe peak. Recompute the SIR.

   Both quantities are again referred to the pattern peak, so the 1.8 dB the main lobe loses cancels in the ratio:

   $$\begin{aligned}\text{SIR} &= 0 \text{ dB} - (10 \text{ dB} - 46 \text{ dB}) \\ &= 36 \text{ dB}\end{aligned}$$

   SIR = 36 dB

   The null adds 33 dB of margin.

   (c) The same 33 dB of margin could also come from raising the transmit power. State what that requires on a radar, in one or two sentences, and say which fix you would specify.

   A radar echo scales with transmit power, so 33 dB more margin requires about 2000 times the transmit power, with the amplifier, prime power, cooling, and transmitted signature that go with it. None of that changes the interferer. Null steering provides the same margin by recomputing sixteen numbers and lowers the main lobe by about 2 dB , so the null is the fix to specify whenever the interferer angle is known.

   (d) The interferer is at $+22.5^\circ$ and the target is on boresight. Explain why the array does not reject the interferer simply because it is off axis.

   An array rejects a direction only by the pattern level it presents in that direction. Being off axis is not enough; the interferer arrives through whatever sidelobe covers $+22.5^\circ$, and for a uniform eight-element array that sidelobe is only 13 dB down.

2. **Steering vectors.** The PHASER array has $N = 8$ elements at $d = 14$ mm, receiving the HB100 tone at 10.525 GHz so that $d/\lambda = 0.491$ and $kd = 3.085$ rad. The steering vector for arrival angle θ has entries $v_n(\theta) = e^{jnkd\sin(\theta)}$, and the beamformer response is $y(\theta) = \sum_n w_n v_n(\theta)$.

   (a) In one or two sentences, state what $\vec{v}(\theta)$ describes and what it does *not* depend on.

   The steering vector describes how a plane wave from direction θ is spread in phase across the aperture: element n leads element 0 by $nkd\sin(\theta)$. It is a property of the arrival direction and the array geometry only, and does not depend on the gains and phases loaded into the beamformer.

   (b) Compute the element-to-element phase for a commanded beam at $\theta_0 = 30°$.

   $$\Delta\phi = kd\sin(\theta_0) = 3.085\,(0.500)$$
   $$= 1.543 \text{ rad} = 88.4°$$

   $\Delta\phi$ = $88.4°$

   (c) Write the weight w_3 (the fourth element, $n = 3$) that steers the beam to $30°$, as a magnitude and an angle.

   Beam steering uses the conjugate steering vector, $w_n = e^{-jnkd\sin(\theta_0)}$, so every element runs at full gain and element n lags by $n\Delta\phi$:

   $$w_3 = e^{-j3(88.4°)} = 1\angle -265.2° = 1\angle +94.8°$$

   w_3 = $1\angle +94.8°$

   (d) With those weights loaded, evaluate $y(30°)$ and express the result in dB relative to one element.

   Each term becomes $e^{-jnkd\sin(\theta_0)}e^{+jnkd\sin(\theta_0)} = 1$, so the eight phasors add in phase:

   $$y(30°) = \sum_{n=0}^{7} 1 = 8$$
   $$20\log_{10}(8) = 18.1 \text{ dB}$$

   y = $8\,(18.1$ dB $)$

3. **Weight subtraction by hand.** A four-element array with $d = \lambda/2$ (so $kd = 180^\circ$) is steered broadside, giving $\vec{w}_\mathrm{d} = [1, 1, 1, 1]$. An interferer appears at $\theta_1 = 45^\circ$. Use the weight-subtraction rule

$$\vec{w} = \vec{w}_\mathrm{d} - r_\mathrm{n}\vec{w}_\mathrm{n}, \qquad r_\mathrm{n} = \frac{\vec{w}_\mathrm{n}^H \vec{w}_\mathrm{d}}{\vec{w}_\mathrm{n}^H \vec{w}_\mathrm{n}}.$$

(a) Compute the element-to-element phase $\psi_1 = kd\sin(\theta_1)$ at the interferer angle.

ψ_1 = 127.3°

$$\psi_1 = 180^\circ \times \sin(45^\circ)$$
$$= 180^\circ(0.7071) = 127.3^\circ$$

(b) Evaluate r_n. For a broadside beam it reduces to the normalized uniform array factor at θ_1,

$$r_\mathrm{n} = e^{j(N-1)\psi_1/2}\,\frac{\sin(N\psi_1/2)}{N\sin(\psi_1/2)}.$$

r_n = $0.269\angle 10.9^\circ$

With $N = 4$ and $\psi_1 = 127.3^\circ$:

$$\frac{\sin(2\psi_1)}{4\sin(\psi_1/2)} = \frac{\sin(254.6^\circ)}{4\sin(63.6^\circ)} = \frac{-0.964}{3.585} = -0.269$$
$$r_\mathrm{n} = (-0.269)\,e^{j190.9^\circ} = 0.269\angle 10.9^\circ$$

The magnitude 0.269 is -11.4 dB , which is the uniform four-element pattern level at 45°.

(c) Compute the four weights $w_n = 1 - r_\mathrm{n}e^{-jn\psi_1}$ and convert them to PHASER-style settings: element gains as percentages of the largest, and phases in degrees.

The subtracted phasor has magnitude 0.269 and angle $10.9^\circ - n(127.3^\circ)$:

$$w_0 = 1 - 0.269\angle 10.9^\circ = 0.736 - j0.051 = 0.738\angle -4.0^\circ$$
$$w_1 = 1 - 0.269\angle -116.4^\circ = 1.119 + j0.241 = 1.145\angle +12.1^\circ$$
$$w_2 = 1 - 0.269\angle -243.7^\circ = 1.119 - j0.241 = 1.145\angle -12.1^\circ$$
$$w_3 = 1 - 0.269\angle -371.0^\circ = 0.736 + j0.051 = 0.738\angle +4.0^\circ$$

Scaling the largest magnitude to 100%, the settings are

$$\text{gains} = 64, 100, 100, 64 \text{ percent}$$
$$\text{phases} = -4.0^\circ,\ +12.1^\circ,\ -12.1^\circ,\ +4.0^\circ$$

The gains are symmetric and the phases antisymmetric, as they must be for a single null placed off a broadside beam.

(d) Find the loss in the look direction after the weights are rescaled so the largest element sits at 100% gain.

loss = 1.8 dB

At broadside every steering-vector entry is 1, so the response is the plain sum of the weights, and the rescaling divides by the largest magnitude:

$$\sum_n w_n = 3.711, \qquad \max_n |w_n| = 1.145$$

$$\text{loss} = 20\log_{10}\left(\frac{3.711}{1.145 \times 4}\right) = -1.8 \text{ dB}$$

4. **Main-lobe loss.** Return to the eight-element PHASER array ($kd = 3.085$ rad, broadside beam). Before the weights are rescaled, the response in the look direction falls from N to $N\left(1 - |r_n|^2\right)$.

   (a) An interferer sits at $\theta_1 = 10°$, where the uniform pattern level is -8.0 dB . Compute the loss in the look direction before rescaling.

   loss = 1.5 dB

   A level of -8.0 dB means $|r_n| = 10^{-8.0/20} = 0.398$:

   $$\text{loss} = 20\log_{10}\left(1 - (0.398)^2\right)$$
   $$= 20\log_{10}(0.842) = -1.5 \text{ dB}$$

   (b) A second interferer sits at $\theta_1 = 30°$, where the uniform pattern level is -33.9 dB . Compute the loss and comment on the result in one sentence.

   loss = ≈ 0 dB

   Here $|r_n| = 10^{-33.9/20} = 0.020$:

   $$\text{loss} = 20\log_{10}\left(1 - (0.020)^2\right) = -0.004 \text{ dB}$$

   The interferer is already sitting in one of the array's natural pattern nulls, so the rule subtracts almost nothing and leaves the weights nearly unchanged.

   (c) The PHASER's half-power beamwidth is $13.1°$. A student asks the array to null an interferer at $\theta_1 = 3°$, where $|r_n| = 0.93$. Explain what the resulting pattern looks like and what the student should do instead.

   With $|r_n| = 0.93$ the rule subtracts almost the entire beam. The eight weights very nearly cancel, and after rescaling what remains is a split pattern with a hole where the target used to be; the look-direction gain falls by more than 12 dB relative to the uniform array. Weighting cannot separate an interferer inside the half-power beamwidth from the target, so the student should steer the beam to a different angle or wait for the geometry to change.

   (d) Explain, in one or two sentences, why $|r_n|$ is worth computing before calculating any weights.

   $|r_n|$ is the level of the pattern you already have at the null angle, and it sets the main-lobe loss. Reading it off the existing pattern tells you whether the null will lower the main lobe by a fraction of a decibel or by many decibels, before any weight arithmetic.

5. **How deep can the null be?** Each ADAR1000 phase setting lands on a 2.8125° grid and each gain on a 1% grid, so each weight carries an independent error. The errors add in RMS at the null angle, giving a residual response

$$\frac{|y(\theta_1)|}{N} \approx \frac{\epsilon_{\text{rms}}}{\sqrt{N}}, \qquad \epsilon_{\text{rms}} = \sqrt{\sigma_\phi^2 + \sigma_a^2}, \qquad \sigma_\phi = \frac{\text{LSB}}{\sqrt{12}},$$

with σ_ϕ in radians.

(a) Compute the quantization-limited null depth for the eight-element PHASER with its 7-bit phase shifters.

depth = -45.8 dB

$$\sigma_\phi = \frac{2.8125^\circ}{\sqrt{12}} = 0.812^\circ = 0.0142 \text{ rad}$$
$$\sigma_a = \frac{0.01}{\sqrt{12}} = 0.0029$$
$$\epsilon_{\text{rms}} = \sqrt{0.0142^2 + 0.0029^2} = 0.0145$$
$$\text{depth} = 20\log_{10}\left(\frac{0.0145}{\sqrt{8}}\right) = -45.8 \text{ dB}$$

The estimate is relative to the null-steered main lobe, which sits 2 dB below the uniform peak, so it is about -48 dB relative to the uniform peak.

(b) Repeat for a 3-bit phase shifter, ignoring the gain error. Compare your answer with the $-6B$ rule of thumb from L23.

depth = -21.9 dB

A 3-bit part has LSB $= 360^\circ/8 = 45^\circ$:

$$\sigma_\phi = \frac{45^\circ}{\sqrt{12}} = 13.0^\circ = 0.227 \text{ rad}$$
$$\text{depth} = 20\log_{10}\left(\frac{0.227}{\sqrt{8}}\right) = -21.9 \text{ dB}$$

The $-6B$ rule predicts -18 dB for three bits, so the null floor lands a few decibels deeper because the eight elements average the error.

(c) On a course kit, the lab sweep measured the notch 46 to 50 dB below the uniform-taper peak, with the null-steered main lobe 2 dB below that same reference. Compare the measurement with your answer to (a) and state what limits the depth.

Referred to the uniform peak, the estimate in (a) is -45.8 dB $-$ 2 dB $\approx$ -48 dB, inside the measured band of -46 to -50 dB. The bench null is as deep as independent errors of about one step per channel allow. Rounding the weights alone does less: it keeps the gains symmetric and the phases antisymmetric, so the pattern still passes through zero near the design angle. The errors that fill the null are each channel's own calibration errors in gain and phase, which are about one step in size.

(d) State the maximum number of independent nulls an eight-element array can place and explain the counting in one sentence.

Seven. The array has eight complex weights, one of which holds the main beam in the look direction, leaving $N - 1 = 7$ degrees of freedom for nulls.

Documentation: _____