Learning Objective 3.8: I can identify grating lobes, beam squint, and quantization effects in a phased array and describe their impact on system performance.

Show your work. A **Documentation** line at the end is required (write **None** if you did not collaborate).

1. **Where the array factor repeats.** The ADALM-PHASER carries 8 patch elements on a 14 mm pitch. A thinning demonstration feeds only every third element, so the fed elements sit 42 mm apart. Work at the array design center, 10.3 GHz, where $\lambda = 29.1$ mm.

   (a) In one or two sentences, explain why a grating lobe reaches the same height as the main lobe, while a sidelobe does not.

   (b) With the beam at broadside, find every grating-lobe angle in visible space.

   θ_g =

   (c) Now steer the beam to $\theta_0 = +20^\circ$. Find every grating lobe that remains in visible space.

   θ_g =

   (d) A classmate proposes applying a Hann taper to remove the extra beams. Explain in one or two sentences why that does not work.

2. **Choosing the element spacing.** You are laying out an X-band linear array. Use $c = 3 \times 10^8$ m/s throughout.

   (a) State the criterion that keeps grating lobes out of visible space over a scan range $\pm\theta_0$, and explain why half-wavelength spacing is the usual default.

   (b) The array must scan to $\pm 60^\circ$ and operate up to 10.5 GHz. Find the largest usable element spacing in millimeters.

   d_{max} =

   (c) A competing layout uses $d = 20$ mm. At 10.5 GHz, how far off broadside can that array scan before a grating lobe enters visible space?

   $\theta_{0,\text{max}}$ =

   (d) The PHASER uses $d = 14$ mm. State its scan limit from the same criterion, and give one sentence on what the design gives up by choosing a spacing this tight.

3. **Beam squint across a band.** An 8-element array with $d = 14$ mm operates from 10.0 to 10.5 GHz. The beam-steering computer sets the element phases for $\theta_0 = 45^\circ$ at the band center, $f_0 = 10.25$ GHz. Use $c = 3 \times 10^8$ m/s.

   (a) Explain in one or two sentences why a broadside beam does not squint at all.

   (b) Find the beam angle at the low band edge, 10.0 GHz.

   θ = [ ]

   (c) Find the beam angle at the high band edge, 10.5 GHz, and state the total angular walk of the beam across the band.

   walk = [ ]

   (d) The half-power beamwidth of this array at 45° is $\theta_{\text{HP}} \approx 0.886\lambda/(Nd\cos(\theta_0))$. Express the walk from part (c) as a fraction of a beamwidth, and recommend a fix if the requirement were to hold the walk under 2% of a beamwidth.

   walk / θ_{HP} = [ ]

4. **Phase-Shifter Resolution.** The ADAR1000 beamformer on the PHASER uses a B-bit phase shifter. Work at the HB100 source frequency, 10.525 GHz, where $\lambda = 28.5$ mm and $d/\lambda = 0.491$.

   (a) Find the phase step (LSB) for $B = 6$ and for the ADAR1000's $B = 7$.

   $B = 6$:

   $B = 7$:

   (b) With $B = 7$, find the pointing step near broadside and at $\theta_0 = 60^\circ$.

   broadside:

   at 60°:

   (c) Use the rule of thumb QSLL $\approx -6B$ dB for $B = 3$ and $B = 7$, and state which sidelobe mechanism dominates in each case for a uniform 8-element array.

   $B = 3$:

   $B = 7$:

   (d) A classmate argues that because the phase shifter resolves only 2.8125°, a pattern null on the PHASER can never go much deeper than 20 dB. Explain in two or three sentences why that reasoning is wrong, and state what does set the measured depth.

5. **Reading a sweep and sizing a design.** Each part below reports something a student measured on the PHASER, or asks for a design decision. The array is 8 elements on a 14 mm pitch unless stated otherwise.

   (a) A broadside sweep against the HB100 at 10.525 GHz ($\lambda = 28.5$ mm) shows three peaks of equal height, at 0° and $\pm 42^\circ$. What effective element spacing does that indicate, and how were the elements fed?

   d_{eff} =

   (b) The student commands the array to $+45^\circ$ with phases set at 10.525 GHz, but the sweep peaks at $+47.9^\circ$ with the source running at 10.025 GHz. Name the effect and confirm the measurement.

   θ =

   (c) At $B = 2$ the sweep shows a lobe near -55° about 7 dB below the peak, while the rule of thumb predicts -12 dB. Explain the discrepancy in one or two sentences.

   (d) A new array must scan to $\pm 50^\circ$ over 10.0 to 10.5 GHz and hold every sidelobe below -25 dB. State the maximum element spacing and the minimum number of phase bits, and say which of the three defects the bandwidth requirement drives.

   d_{max} =

   B_{min} =

Documentation: _____