Learning Objective 3.7: I can apply amplitude tapering (uniform, cosine, Chebyshev, Taylor) to control sidelobe level and predict the pattern trade-off.

Show your work. A **Documentation** line at the end is required (write **None** if you did not collaborate).

1. **Where sidelobes come from, and why a system cares.** The PHASER drives its eight elements at equal amplitude, spaced $d = 14$ mm apart, which is $d/\lambda = 0.481$ at 10.3 GHz. Its measured first sidelobe sits 12.8 dB below the peak.

   (a) In two or three sentences, explain why a uniformly illuminated aperture has sidelobes at all. Use the Fourier relationship between the aperture illumination and the far-field pattern in your answer.

   (b) The array is rebuilt with $N = 16$ elements at the same spacing, still uniformly driven. Compute the new half-power beamwidth from $\theta_{\text{HP}} \approx 0.886\lambda/(Nd)$, and state what happens to the first sidelobe level.

   θ_{HP} = ____

   (c) A search radar looks at a target on boresight while ground clutter 40 dB stronger than the target return arrives through the -12.8 dB first sidelobe. Compute the clutter-to-target ratio at the beamformer output, then find the sidelobe level that would put the clutter 3 dB *below* the target.

   C/T = ____

   SLL = ____

2. **The Hann preset by the numbers.** The PHASER GUI's Hann preset sets the eight Element Gains to 12, 43, 77, 100, 100, 77, 43, 12 percent. Work in normalized amplitudes a_n of 0.12, 0.43, 0.77, 1.00, 1.00, 0.77, 0.43, 0.12.

   (a) Compute $\sum a_n$ and $\sum a_n^2$.

   $\sum a_n$ =

   $\sum a_n^2$ =

   (b) Compute the taper efficiency

   $$\eta_t = \frac{\left(\sum a_n\right)^2}{N \sum a_n^2}$$

   η_t =

   as a ratio and in decibels.

   (c) Compute how far the peak of the measured sweep falls below the uniform-taper trace.

   peak drop =

   (d) Parts (b) and (c) are both decibel losses, and they are not equal. In two or three sentences, say what each one measures and which of the two is the loss of directivity.

3. **A Chebyshev design to specification.** A customer specifies that no sidelobe may exceed -25 dB relative to the peak, on the eight-element array at $d/\lambda = 0.481$. The uniform reference is $\theta_{\text{HP}} = 13.2^\circ$.

   (a) Compute the voltage ratio R and the Chebyshev stretch factor

   $$x_0 = \cosh\left(\frac{\cosh^{-1}(R)}{N-1}\right).$$

   R =

   x_0 =

   The Dolph expansion for this x_0 returns element amplitudes of $38, 58, 84, 100, 100, 84, 58, 38$ percent.

   (b) Compute the taper efficiency and the peak drop for this design.

   η_t =

   peak drop =

   (c) This design has a beam broadening factor of 1.21. Compute its half-power beamwidth.

   θ_{HP} =

   (d) State what this design gives up for the customer in one sentence, then explain in one or two more why a Chebyshev design beats a Hann window that reaches a similar sidelobe level.

4. **Reading a tapered sweep.** A student loads an unknown taper into the GUI and runs a beam sweep against the HB100. Compared with the uniform trace taken a minute earlier, the peak is 3.8 dB lower, the half-power beamwidth reads 17.5°, and no sidelobe is visible above the sweep's noise floor, which sits 23 dB below the *uniform* peak.

   (a) Which of the designs from this lesson is the student most likely using? Give the two measurements that identify it.

   (b) How much directivity did the array lose? The design has $\sum a_n = 5.18$ and $\sum a_n^2 = 3.988$.

   loss = ____

   (c) The student writes in the lab notebook that the array lost 3.8 dB of gain. Correct that statement in two sentences.

   (d) State the tightest upper bound the student can put on this taper's sidelobe level, expressed relative to the taper's own peak.

   SLL $<$ ____

5. **Design judgment on eight elements.** A program office asks for -50 dB sidelobes from the eight-element PHASER array. The Chebyshev design for it has element amplitudes $9, 35, 72, 100, 100, 72, 35, 9$ percent, a beam broadening factor of 1.53, and $\eta_t = 0.710$.

   (a) Compute the half-power beamwidth and the peak drop for this design, using 13.2° as the uniform reference.

   θ_{HP} = ____

   peak drop = ____

   (b) The ADAR1000 sets each element gain to a resolution of about one percent of full scale. Explain in two or three sentences why that resolution threatens a -50 dB design more than it threatens a -25 dB design.

   (c) Recommend what to do about the program office's request. Your answer must name the specification you would propose instead and give the reason in terms of what the hardware can show.

Documentation: _____