Learning Objective 3.7: I can apply amplitude tapering (uniform, cosine, Chebyshev, Taylor) to control sidelobe level and predict the pattern trade-off.

Show your work. A **Documentation** line at the end is required (write **None** if you did not collaborate).

1. **Where sidelobes come from, and why a system cares.** The PHASER drives its eight elements at equal amplitude, spaced $d = 14$ mm apart, which is $d/\lambda = 0.481$ at 10.3 GHz. Its measured first sidelobe sits 12.8 dB below the peak.

   (a) In two or three sentences, explain why a uniformly illuminated aperture has sidelobes at all. Use the Fourier relationship between the aperture illumination and the far-field pattern in your answer.

   The far-field pattern is the Fourier transform of the aperture illumination, with the transform variable being the space frequency $k\sin(\theta)$. Uniform illumination is a rectangle: full amplitude across the array and nothing beyond the ends. That abrupt edge is a discontinuity, and a discontinuity contains energy at every spatial frequency, so the transform cannot fall off quickly. The transform of a rectangle is a sinc function, whose ripples decay only as $1/u$, and those ripples are the sidelobes.

   (b) The array is rebuilt with $N = 16$ elements at the same spacing, still uniformly driven. Compute the new half-power beamwidth from $\theta_{\text{HP}} \approx 0.886\lambda/(Nd)$, and state what happens to the first sidelobe level.

   θ_{HP} = 6.6°

   $$\theta_{\text{HP}} \approx \frac{0.886}{16 \times 0.481} = 0.1151 \text{ rad} = 6.6^\circ$$

   The first sidelobe does not move: it stays near -13 dB . Doubling the aperture halves the beamwidth, but sidelobe level is set by the *shape* of the illumination, not by its size, and the shape is still uniform.

   (c) A search radar looks at a target on boresight while ground clutter 40 dB stronger than the target return arrives through the -12.8 dB first sidelobe. Compute the clutter-to-target ratio at the beamformer output, then find the sidelobe level that would put the clutter 3 dB *below* the target.

   C/T = 27.2 dB

   SLL = -43 dB

   The target enters on the main beam at 0 dB of pattern gain and the clutter enters 12.8 dB down, so the antenna gives the target a 12.8 dB advantage:

   $$C/T = 40 - 12.8 = 27.2 \text{ dB (clutter above target)}$$

   To land at $C/T = -3$ dB the pattern must supply 43 dB of discrimination:

   $$\text{SLL} = -(40 + 3) = -43 \text{ dB}$$

2. **The Hann preset by the numbers.** The PHASER GUI's Hann preset sets the eight Element Gains to $12, 43, 77, 100, 100, 77, 43, 12$ percent. Work in normalized amplitudes a_n of $0.12, 0.43, 0.77, 1.00, 1.00, 0.77, 0.43, 0.12$.

   (a) Compute $\sum a_n$ and $\sum a_n^2$.

   The taper is symmetric, so sum the inner four and double:

   $$\sum a_n = 2\,(0.12 + 0.43 + 0.77 + 1.00) = 2\,(2.32) = 4.64$$
   $$\sum a_n^2 = 2\,(0.0144 + 0.1849 + 0.5929 + 1.0000) = 3.584$$

   $\sum a_n$ = 4.64

   $\sum a_n^2$ = 3.584

   (b) Compute the taper efficiency

   $$\eta_t = \frac{\left(\sum a_n\right)^2}{N \sum a_n^2}$$

   η_t = $0.751 = -1.2$ dB

   as a ratio and in decibels.

   $$\eta_t = \frac{(4.64)^2}{8 \times 3.584} = \frac{21.53}{28.67} = 0.751$$
   $$10\log_{10}(0.751) = -1.24 \text{ dB}$$

   (c) Compute how far the peak of the measured sweep falls below the uniform-taper trace.

   $$\text{peak drop} = 20\log_{10}\left(\frac{\sum a_n}{N}\right) = 20\log_{10}\left(\frac{4.64}{8}\right)$$
   $$= 20\log_{10}(0.580) = -4.7 \text{ dB}$$

   peak drop = -4.7 dB

   (d) Parts (b) and (c) are both decibel losses, and they are not equal. In two or three sentences, say what each one measures and which of the two is the loss of directivity.

   The taper efficiency in part (b) is the fraction of the uniform array's directivity that the tapered array keeps, so -1.2 dB is the loss of antenna performance. The peak drop in part (c) is a coherent receive-voltage loss: the taper attenuates six of the eight channels, so the plotted peak fell, but it attenuates their noise along with the signal. The difference, $-4.7 - (-1.2) = -3.5$ dB, is gain that was simply turned down and would return if the sliders could be scaled back up past 100%.

3. **A Chebyshev design to specification.** A customer specifies that no sidelobe may exceed -25 dB relative to the peak, on the eight-element array at $d/\lambda = 0.481$. The uniform reference is $\theta_{\text{HP}} = 13.2^\circ$.

   (a) Compute the voltage ratio R and the Chebyshev stretch factor

   $$x_0 = \cosh\left(\frac{\cosh^{-1}(R)}{N-1}\right).$$

   $R =$ 17.78

   $x_0 =$ 1.133

   $$R = 10^{25/20} = 10^{1.25} = 17.78$$
   $$\cosh^{-1}(17.78) = 3.571$$
   $$x_0 = \cosh\left(\frac{3.571}{7}\right) = \cosh(0.5101)$$
   $$= 1.133$$

   The Dolph expansion for this x_0 returns element amplitudes of $38, 58, 84, 100, 100, 84, 58, 38$ percent.

   (b) Compute the taper efficiency and the peak drop for this design.

   $\eta_t =$ $0.896 = -0.48$ dB

   peak drop = -3.1 dB

   $$\sum a_n = 2(0.38 + 0.58 + 0.84 + 1.00) = 5.60$$
   $$\sum a_n^2 = 2(0.1444 + 0.3364 + 0.7056 + 1)$$
   $$= 4.373$$
   $$\eta_t = \frac{(5.60)^2}{8 \times 4.373} = \frac{31.36}{34.98} = 0.896 = -0.48 \text{ dB}$$
   $$\text{peak drop} = 20\log_{10}\left(\frac{5.60}{8}\right)$$
   $$= 20\log_{10}(0.700) = -3.1 \text{ dB}$$

   (c) This design has a beam broadening factor of 1.21. Compute its half-power beamwidth.

   $\theta_{\text{HP}} =$ 16.0°

   $$\theta_{\text{HP}} = 1.21 \times 13.2^\circ = 16.0^\circ$$

   (d) State what this design gives up for the customer in one sentence, then explain in one or two more why a Chebyshev design beats a Hann window that reaches a similar sidelobe level.

   Sidelobes drop from -12.8 dB to -25 dB, and the design gives up a beam 2.8° wider, 0.48 dB of directivity, and a 3.1 dB lower peak on the displayed trace. The Chebyshev design is optimal in the sense that no other set of amplitudes reaches -25 dB with a narrower main lobe, because it holds every sidelobe exactly at the specification. A Hann window pushes its far sidelobes well below -25 dB, which is suppression the specification never asked for, and it widened the main lobe to get that suppression.

4. **Reading a tapered sweep.** A student loads an unknown taper into the GUI and runs a beam sweep against the HB100. Compared with the uniform trace taken a minute earlier, the peak is 3.8 dB lower, the half-power beamwidth reads $17.5°$, and no sidelobe is visible above the sweep's noise floor, which sits 23 dB below the *uniform* peak.

   (a) Which of the designs from this lesson is the student most likely using? Give the two measurements that identify it.

   The -30 dB Chebyshev design, whose element gains are $26, 52, 81, 100, 100, 81, 52, 26$ percent. It predicts a peak drop of -3.8 dB and a half-power beamwidth of $17.1°$, and the measured -3.8 dB and $17.5°$ match both within the sweep's angular resolution. The Hann preset would have dropped the peak 4.7 dB and widened the beam to about $19.5°$.

   (b) How much directivity did the array lose? The design has $\sum a_n = 5.18$ and $\sum a_n^2 = 3.988$.

   loss = -0.75 dB

$$\eta_t = \frac{(5.18)^2}{8 \times 3.988} = \frac{26.83}{31.91} = 0.841$$

$$10\log_{10}(0.841) = -0.75 \text{ dB}$$

   (c) The student writes in the lab notebook that the array lost 3.8 dB of gain. Correct that statement in two sentences.

   The array lost 0.75 dB of directivity, which is the taper efficiency computed in part (b). The remaining 3.0 dB of the drop is signal that six attenuated channels never delivered to the summing node, and the noise from those channels was attenuated by the same factor, so the sensitivity of the array did not fall by 3.8 dB .

   (d) State the tightest upper bound the student can put on this taper's sidelobe level, expressed relative to the taper's own peak.

   SLL $<$ -19.2 dB

   The floor sits 23 dB below the uniform peak, and this taper's peak is itself 3.8 dB below the uniform peak, so relative to its own peak the floor is only

$$23 - 3.8 = 19.2 \text{ dB down}$$

   The sidelobes are therefore below -19 dB relative to the tapered peak. The theoretical value is -30 dB , but the measurement cannot confirm anything deeper than the floor allows.

5. **Design judgment on eight elements.** A program office asks for -50 dB sidelobes from the eight-element PHASER array. The Chebyshev design for it has element amplitudes $9, 35, 72, 100, 100, 72, 35, 9$ percent, a beam broadening factor of 1.53, and $\eta_t = 0.710$.

   (a) Compute the half-power beamwidth and the peak drop for this design, using 13.2° as the uniform reference.

$$\sum a_n = 2\,(0.09 + 0.35 + 0.72 + 1.00) = 4.32$$

$$\theta_{\text{HP}} = 1.53 \times 13.2^\circ = 20.2^\circ$$

$$\text{peak drop} = 20\log_{10}\left(\frac{4.32}{8}\right)$$

$$= 20\log_{10}(0.540) = -5.3 \text{ dB}$$

θ_{HP} = 20.2°

peak drop = -5.3 dB

   (b) The ADAR1000 sets each element gain to a resolution of about one percent of full scale. Explain in two or three sentences why that resolution threatens a -50 dB design more than it threatens a -25 dB design.

A sidelobe at -50 dB is a residue about three parts in a thousand of the peak, so it survives only if the eight amplitudes hold their designed ratios to better than that. The outermost elements of this design sit at 9%, where a one-percent step is an eleven-percent error on that element, which is two orders of magnitude larger than the residue the design is trying to preserve. The -25 dB design keeps its smallest element at 38%, where the same step is under three percent, so its sidelobes move by a fraction of a decibel instead of being swamped. Rounding to the whole-percent gains in the stem already lifts the -50 dB design's highest sidelobe to -44.3 dB .

   (c) Recommend what to do about the program office's request. Your answer must name the specification you would propose instead and give the reason in terms of what the hardware can show.

Propose -30 dB rather than -50 dB . The beam sweep's noise floor is about 23 dB below the uniform peak, so a -30 dB design is already below anything this instrument can measure, and the bench cannot verify a -50 dB design at all. The deeper specification also widens the beam 3° more and loses roughly 0.7 dB more directivity than the -30 dB design, and its edge elements are too small to set accurately. If the office needs suppression in one specific direction rather than everywhere, the correct tool is a steered null, not a deeper taper.

Documentation: _____