**Learning Objective 3.6:** I can distinguish between array factor and true antenna pattern and account for element pattern effects.

Show your work. A **Documentation** line at the end is required (write **None** if you did not collaborate).

1. **The gain budget of the course array.** The ADALM-PHASER carries $N = 8$ microstrip patch elements spaced $d = 14$ mm apart. At the workshop frequency of $10.3$ GHz the wavelength is $29.1$ mm. Model each element as an ideal element that captures its share of the aperture, so its *power* pattern follows the projected-aperture rule $G_e(\theta) = G_e(0)\cos(\theta)$ with a broadside element gain of $6.0$ dBi.

   (a) In one or two sentences, state the difference between the array factor and the antenna pattern, and say what the element factor contributes to the product.

   (b) Compute the broadside peak gain of the array from the element gain and the array gain.

   $G(0^\circ)$ =

   (c) The beam is steered to $\theta_0 = 45^\circ$. Compute the element factor penalty in decibels at that angle.

   $\Delta G$ =

   (d) Compute the peak gain of the array steered to $45^\circ$.

   $G(45^\circ)$ =

2. **Scan loss and the practical scan limit.** Use the same array and the same ideal element. Recall from L20 that the array factor's half-power beamwidth off broadside is $\theta_{\text{HP}} \approx 0.886\lambda/(Nd\cos(\theta_0))$, which is $13.2^\circ$ at broadside for this array.

   (a) Compute the element factor penalty at $\theta_0 = 30^\circ$.

   $\Delta G$ = [ ]

   (b) Compute the element factor penalty at $\theta_0 = 60^\circ$.

   $\Delta G$ = [ ]

   (c) Compute the array factor half-power beamwidth at $\theta_0 = 60^\circ$.

   $\theta_{\text{HP}}$ = [ ]

   (d) A radar engineer specifies this array for coverage over $\pm 75^\circ$. Using your numbers from parts (a) through (c), write two sentences explaining the trade the engineer is accepting, and state why $\pm 60^\circ$ is the usual limit quoted for patch arrays.

3. **Reading a steered sweep.** A student steers the array to $\theta_0 = 30^\circ$ and records the rectangular sweep. The peak reads $-0.6$ dB relative to the broadside peak and sits at $29.6^\circ$. The first sidelobe on the broadside side of the beam reads $-12.2$ dBc and the first sidelobe on the outboard side reads $-15.2$ dBc. Uniform array factor theory predicts $-12.8$ dBc for both.

   (a) Explain in one or two sentences why the measured peak sits at $29.6^\circ$ rather than at the commanded $30^\circ$.

   (b) Explain why the two first sidelobes differ by $3$ dB when the array factor predicts that they are equal.

   (c) Express the inboard sidelobe level relative to the array's *broadside* peak rather than relative to the steered peak.

   Level = [ ]

   (d) The same student holds the beam fixed, carries the source around the array by hand, and repeats the measurement. The new trace has the same shape but every feature is shifted $1.8^\circ$ in the same direction. State whether this is an antenna effect or a measurement effect, and give the reason.

4. **The element and its ground plane.** Take the ideal element power pattern to be $\cos(\theta)$ over the front hemisphere and zero behind the ground plane.

   (a) Find the half-power beamwidth of this element.

   $\theta_{\text{HP}} =$ ____

   (b) Compute the directivity of this element, in dBi, integrating over the upper hemisphere only.

   $D_e =$ ____

   (c) A sweep shows a lobe at $-20$ dBc near $\theta = 82^\circ$. Explain what this feature is, and why it cannot be radiation out the back of the array.

   (d) The projected-aperture rule is a model. State one angular range where you would trust a prediction made with it to about a decibel, and one where you would not, and give the reason.

5. **Range, environment, and what a measurement can resolve.** The course array measures $D = 98$ mm between the centers of its outer elements. You place the lab source $1.2$ m from the array face.

   (a) Compute the far-field distance for this array at $10.3$ GHz ($\lambda = 29.1$ mm), and state whether the lab range qualifies.

   $r$ =

   (b) You need to measure a larger array on the same $1.2$ m range at $10.3$ GHz. Find the largest aperture dimension that still satisfies the far-field criterion.

   $D$ =

   (c) The GUI sweeps the beam by stepping the element-to-element phase one $2.8125^\circ$ LSB per point, about $0.93^\circ$ of angle near broadside. Explain how that step size alone can make a measured half-power beamwidth differ from theory by up to about one degree.

   (d) Name two things you can change about the measurement setup, not the antenna, to make the far-out sidelobes of a trace more trustworthy.

**Documentation:** _____