Learning Objective 3.6: I can distinguish between array factor and true antenna pattern and account for element pattern effects.

Show your work. A **Documentation** line at the end is required (write **None** if you did not collaborate).

1. **The gain budget of the course array.** The ADALM-PHASER carries $N = 8$ microstrip patch elements spaced $d = 14$ mm apart. At the workshop frequency of 10.3 GHz the wavelength is 29.1 mm. Model each element as an ideal element that captures its share of the aperture, so its *power* pattern follows the projected-aperture rule $G_e(\theta) = G_e(0)\cos(\theta)$ with a broadside element gain of 6.0 dBi.

   (a) In one or two sentences, state the difference between the array factor and the antenna pattern, and say what the element factor contributes to the product.

   The array factor is the contribution of the element positions and their relative phases alone; it is what a set of isotropic radiators at those positions would produce. The antenna pattern is the array factor multiplied by the element factor, which is the pattern of one element in the array. The element factor supplies the shape of a single radiator – here a broad hemispherical lobe peaked at broadside – and it sets an envelope that the steered array factor cannot exceed.

   (b) Compute the broadside peak gain of the array from the element gain and the array gain.

   The eight elements add coherently, so the array gain is

   $$G_{\text{array}} = 10\log_{10}(8) = 9.0 \text{ dB}$$

   and the total broadside gain is

   $$G(0^\circ) = 6.0 \text{ dBi} + 9.0 \text{ dB} = 15.0 \text{ dBi}$$

   $G(0^\circ)$ = **15.0 dBi**

   (c) The beam is steered to $\theta_0 = 45^\circ$. Compute the element factor penalty in decibels at that angle.

   The projected-aperture rule is stated in power, so the penalty uses ten times the logarithm:

   $$10\log_{10}(\cos(45^\circ)) = 10\log_{10}(0.707) = -1.5 \text{ dB}$$

   ΔG = **−1.5 dB**

   (d) Compute the peak gain of the array steered to 45°.

   Only the element term depends on the steer angle; the array gain of 9.0 dB is unchanged because the elements still add in phase.

   $$G(45^\circ) = 6.0 \text{ dBi} - 1.5 \text{ dB} + 9.0 \text{ dB} = 13.5 \text{ dBi}$$

   $G(45^\circ)$ = **13.5 dBi**

2. **Scan loss and the practical scan limit.** Use the same array and the same ideal element. Recall from L20 that the array factor's half-power beamwidth off broadside is $\theta_{\text{HP}} \approx 0.886\lambda/(Nd\cos(\theta_0))$, which is 13.2° at broadside for this array.

   (a) Compute the element factor penalty at $\theta_0 = 30^\circ$.

   $$10\log_{10}(\cos(30^\circ)) = 10\log_{10}(0.866) = -0.6 \text{ dB}$$

   $\Delta G =$ | -0.6 dB |

   (b) Compute the element factor penalty at $\theta_0 = 60^\circ$.

   $$10\log_{10}(\cos(60^\circ)) = 10\log_{10}(0.500) = -3.0 \text{ dB}$$

   $\Delta G =$ | -3.0 dB |

   This is the ideal-element bound. A real patch rolls off faster, roughly as $\cos(\theta)^{1.4}$ in power, and measures about -4.2 dB at the same angle.

   (c) Compute the array factor half-power beamwidth at $\theta_0 = 60^\circ$.

   $$\theta_{\text{HP}} = \frac{13.2^\circ}{\cos(60^\circ)} = \frac{13.2^\circ}{0.500} = 26.4^\circ$$

   $\theta_{\text{HP}} =$ | 26.4° |

   (d) A radar engineer specifies this array for coverage over $\pm 75^\circ$. Using your numbers from parts (a) through (c), write two sentences explaining the trade the engineer is accepting, and state why $\pm 60^\circ$ is the usual limit quoted for patch arrays.

   At 60° the beam has already lost 3 dB of gain for an ideal element, closer to 4 dB for a real patch, and the array factor's beam has doubled in width from 13.2° to 26.4°, so the same target returns less power and is located less precisely. Pushing to 75° loses about 5.9 dB and widens the beam to nearly 51°, so a target at the edge of the scan is detected at shorter range and located far less precisely than one at broadside. The $\pm 60^\circ$ limit is where both penalties are still tolerable; beyond it the gain falls faster than the coverage grows.

3. **Reading a steered sweep.** A student steers the array to $\theta_0 = 30^\circ$ and records the rectangular sweep. The peak reads -0.6 dB relative to the broadside peak and sits at 29.6°. The first sidelobe on the broadside side of the beam reads -12.2 dBc and the first sidelobe on the outboard side reads -15.2 dBc. Uniform array factor theory predicts -12.8 dBc for both.

   (a) Explain in one or two sentences why the measured peak sits at 29.6° rather than at the commanded 30°.

   The element factor falls monotonically away from broadside, so the product of the array factor and the element factor is largest slightly inboard of the array factor's own peak. The beam is pulled toward broadside. The effect grows with steer angle: a 60° command on this array peaks near 56.8°.

   (b) Explain why the two first sidelobes differ by 3 dB when the array factor predicts that they are equal.

   Sidelobe levels are quoted relative to the peak of the same trace, and the element factor multiplies each lobe by a different amount. The inboard lobe near 7° sits where the element pattern is essentially flat, so it is unchanged, while the peak it is referenced to fell by 0.6 dB; that lobe therefore reads about half a decibel *higher* than theory. The outboard lobe near 60° is attenuated 3.0 dB by the element pattern, so it reads well below theory.

   (c) Express the inboard sidelobe level relative to the array's *broadside* peak rather than relative to the steered peak.

   The steered peak is 0.6 dB below the broadside peak, so

   $$-12.2\ \text{dB} - 0.6\ \text{dB} = -12.8\ \text{dB}$$

   relative to broadside. Referenced this way the lobe matches the array factor prediction exactly, which shows that nothing about the sidelobe changed – only the reference did.

   Level = $\boxed{-12.8\ \text{dB}}$

   (d) The same student holds the beam fixed, carries the source around the array by hand, and repeats the measurement. The new trace has the same shape but every feature is shifted 1.8° in the same direction. State whether this is an antenna effect or a measurement effect, and give the reason.

   It is a measurement effect. A uniform shift of every feature by the same amount, with no change in shape, is an offset in the angle axis, from the taped marks or from where the source sat at boresight. An antenna effect such as beam pulling changes the peak and the sidelobes by different amounts and therefore changes the shape of the trace, not just its position.

4. **The element and its ground plane.** Take the ideal element power pattern to be $\cos(\theta)$ over the front hemisphere and zero behind the ground plane.

   (a) Find the half-power beamwidth of this element.

   The pattern is stated in power, so half power is where

   $$\cos(\theta) = 0.5 \quad \rightarrow \quad \theta = 60^\circ$$

   and the full beamwidth is twice that, 120°. A real patch is steeper and measures nearer 105°.

   $\theta_{\text{HP}} =$ $\boxed{120^\circ}$

   (b) Compute the directivity of this element, in dBi, integrating over the upper hemisphere only.

   The beam solid angle is

   $$\Omega_A = \int_0^{2\pi} \int_0^{\pi/2} \cos(\theta) \sin\theta \, d\theta \, d\phi$$
   $$= 2\pi \left[\frac{\sin^2\theta}{2} \right]_0^{\pi/2} = \pi$$

   so

   $$D_e = \frac{4\pi}{\Omega_A} = \frac{4\pi}{\pi} = 4 = 6.0 \text{ dBi}$$

   A real patch rolls off as $\cos(\theta)^n$ with n near 1.3 to 1.5; the same integral gives $D_e = 2(n+1)$, or 6.6 to 7.0 dBi. The steeper element concentrates its power toward broadside, so its directivity is higher even though its gain at wide angles is lower.

   $D_e =$ $\boxed{4\,(6.0 \text{ dBi})}$

   (c) A sweep shows a lobe at -20 dBc near $\theta = 82^\circ$. Explain what this feature is, and why it cannot be radiation out the back of the array.

   The ground plane removes the back hemisphere, and the front-hemisphere element pattern falls monotonically toward the horizon, so the antenna itself cannot produce a lobe there. The feature is energy arriving from a reflection – a wall, a bench, or a person – reaching the array from a direction the source is not in. Moving the fixture or the reflector changes the feature, which is the test that identifies it.

   (d) The projected-aperture rule is a model. State one angular range where you would trust a prediction made with it to about a decibel, and one where you would not, and give the reason.

   Inside about $\pm 30^\circ$ the prediction is good to a few tenths of a decibel, because real embedded elements roll off between $\cos(\theta)$ and $\cos(\theta)^{1.5}$ in power and the choices barely differ there. Past about $\pm 60^\circ$ that same spread exceeds a decibel – the ideal rule gives -3.0 dB where a real patch gives about -4.2 dB – and mutual coupling makes each element's embedded pattern differ from the average, so a single-element model should not be trusted to a decibel at wide scan.

5. **Range, environment, and what a measurement can resolve.** The course array measures $D = 98$ mm between the centers of its outer elements. You place the lab source 1.2 m from the array face.

   (a) Compute the far-field distance for this array at 10.3 GHz ($\lambda = 29.1$ mm), and state whether the lab range qualifies.

   $r =$ 0.66 m

   $$r \geqslant \frac{2D^2}{\lambda} = \frac{2\,(0.098\text{ m})^2}{0.0291\text{ m}} = 0.66\text{ m}$$

   The 1.2 m lab range is comfortably beyond this, so the incoming wavefront is flat enough that the edge-to-center phase error stays under the $22.5°$ the criterion allows.

   (b) You need to measure a larger array on the same 1.2 m range at 10.3 GHz. Find the largest aperture dimension that still satisfies the far-field criterion.

   $D =$ 132 mm

   Solve the criterion for D:

   $$D = \sqrt{\frac{r\lambda}{2}} = \sqrt{\frac{(1.2\text{ m})\,(0.0291\text{ m})}{2}} = 0.132\text{ m}$$

   so an aperture larger than about 132 mm needs a longer range or a near-field correction.

   (c) The GUI sweeps the beam by stepping the element-to-element phase one $2.8125°$ LSB per point, about $0.93°$ of angle near broadside. Explain how that step size alone can make a measured half-power beamwidth differ from theory by up to about one degree.

   The trace exists only at grid points, so each half-power crossing is reported at the nearest step rather than at its true angle. Each of the two crossings can be off by up to half a step, and the beamwidth is the difference between them, so the reported width can be off by up to a full step, about $1°$, before any antenna effect is involved.

   (d) Name two things you can change about the measurement setup, not the antenna, to make the far-out sidelobes of a trace more trustworthy.

   Reduce what the environment contributes: place absorber on the bench and on nearby reflecting surfaces, keep people and equipment out of the path, and raise the array and source to the same height away from large flat surfaces. Repeat the measurement with the whole fixture moved or rotated; features that move with the room are environmental, and features that stay with the array are the antenna. Increasing the source power raises the trace above the sweep noise floor, which is what limits any reading more than about 23 dB below the peak.

Documentation: _____