Learning Objective 3.2: I can derive the array factor for an arbitrary linear array and apply pattern multiplication.

Show your work. A **Documentation** line at the end is required (write **None** if you did not collaborate).

1. **Beamwidth of the course array.** The ADALM-PHASER array has $N = 8$ patch elements on $d = 14$ mm centers. For this problem, the source is at 10.3 GHz. Use $\theta_{\text{HP}} \approx 0.886\lambda/\left(Nd\cos\theta_0\right)$ throughout.

   (a) Find the wavelength and the element spacing in wavelengths.

   $d/\lambda =$ ☐

   (b) Compute the half-power beamwidth at broadside.

   $\theta_{\text{HP}} =$ ☐

   (c) Compute the half-power beamwidth with the beam steered to $\theta_0 = 45^\circ$.

   $\theta_{\text{HP}} =$ ☐

   (d) In one or two sentences, explain physically why the beam widens by $1/\cos\theta_0$ when the array is steered, even though no hardware changed.

2. **Nulls and the first-null beamwidth.** Same array as Question 1: $N = 8$, $d/\lambda = 0.481$, so $Nd = 3.85\lambda$. The nulls of a uniform array are at $\sin\theta = \sin\theta_0 \pm m\lambda/(Nd)$ for $m = 1, 2, 3, \ldots$, excluding multiples of N.

   (a) With the beam at broadside, find every null angle on the positive side of the pattern.

   nulls =

   (b) Compute the first-null beamwidth at broadside, and the ratio of it to the half-power beamwidth from Question 1(b).

   FNBW =

   (c) The array factor of an 8-element array has 7 nulls per period, but part (a) found only 3 on each side of broadside. Explain in one or two sentences where the others went.

   (d) Steer the beam to $\theta_0 = 30^\circ$ and find the two nulls that bracket the main lobe, then the first-null beamwidth.

   FNBW =

3. **Directivity of a uniform linear array.** Use $D \approx 2Nd/\lambda$ for a uniform broadside array of isotropic elements.

   (a) Compute the directivity of the 8-element course array, in linear units and in decibels.

   $D =$ [ ]

   (b) Suppose the same 8 elements were half-wave dipoles rather than patches. Estimate the total gain of the array in dBi.

   $G_{\text{total}} =$ [ ]

   (c) Turning off the outer four elements leaves the center four running. State what happens to the directivity and to the half-power beamwidth, with numbers, and explain in one sentence why both changes have the same cause.

4. **Sizing an array.** A ground-based X-band tracker at 10 GHz requires a half-power beamwidth of 2.0° and must steer to $\pm 60^\circ$.

   (a) Find the aperture length required at broadside, in millimeters and in wavelengths.

   $Nd =$ ____

   (b) The grating-lobe criterion is $d < \lambda/(1 + |\sin\theta_0|_{\max})$. Find the largest usable spacing for this scan volume, then the element count if you build the array at $d = \lambda/2$.

   $d_{\max} =$ ____

   $N =$ ____

   (c) A colleague proposes $d = 0.9\lambda$ instead, which reaches the same 25.4λ aperture with 28 elements rather than 51 — a large saving in phase shifters and amplifiers. Evaluate the proposal: compute where the grating lobe lands at the edge of the scan volume, and state whether you would accept it.

   (d) The beamwidth requirement must hold across the whole scan volume, not just at broadside. State the aperture this demands and the resulting element count at $d = \lambda/2$.

5. **Reading a measured sweep.** A student sweeps the course array at 10.3 GHz ($\lambda = 29.1$ mm) with the GUI and records a main lobe 13.1° wide at half power, a null-to-null width of 31.0°, and a first sidelobe at -11.5 dBc.

   (a) From the measured half-power width alone, infer the aperture in millimeters and the number of active elements on 14 mm centers.

   N =

   (b) Theory puts the first-null beamwidth at 30.1° and the student measured 31.0°. The sweep steps the element-to-element phase by the phase-shifter resolution of 2.8125° per point. Explain the discrepancy in one or two sentences.

   (c) The measured first sidelobe is -11.5 dBc, while theory for a uniform 8-element array gives -12.8 dBc. Is this evidence of a fault? Answer in one or two sentences.

   (d) A second sweep on the same kit, with a different set of element gains loaded, shows a main lobe 27° wide with no visible sidelobes above the noise floor. State which aperture configuration this is and what it implies about the number of active elements.

   N =

Documentation: _____