Learning Objective 3.2: I can derive the array factor for an arbitrary linear array and apply pattern multiplication.

Show your work. A **Documentation** line at the end is required (write **None** if you did not collaborate).

1. **Building the sum.** Three isotropic elements sit on a line at $z = 0$, $z = d$, and $z = 2d$ with $d = \lambda/2$. They are fed in phase with amplitude weights $a_0 = 1$, $a_1 = 2$, $a_2 = 1$. We measure the scan angle θ from broadside.

   (a) Pattern multiplication lets you write the array pattern as one element pattern times one sum. State the two conditions the elements must satisfy for that factoring to be valid, and name the step in the derivation that uses each one.

   (b) Write the array factor as a function of $\psi = kd\sin(\theta)$, then find the extra path length and the extra phase that element 2 contributes relative to element 0 when $\theta = 30^\circ$.

   Δr_2 =

   $\Delta\phi_2$ =

   (c) Evaluate $|AF|$ at broadside and at $\theta = 30^\circ$, and give the 30° value in dB relative to the broadside peak.

   $|AF|_{\theta=0}$ =

   Level at 30° =

   (d) The derivation keeps the phase difference between elements but throws away the amplitude difference. Explain in one or two sentences why that is consistent, given a 112 mm array observed at a range of 30 m.

2. **The course array.** The PHASER carries $N = 8$ patch elements on a line with spacing $d = 14$ mm, uniformly excited and unsteered. Work at $f = 10.3$ GHz.

   (a) Find λ, the spacing in wavelengths, and the range of the inter-element phase ψ over the visible region.

   d/λ = [ ]

   ψ range = [ ]

   (b) Locate every null of the array factor that falls in the visible region.

   Nulls = [ ]

   (c) Find the first-null beamwidth and the half-power beamwidth, and state the first sidelobe level you expect.

   FNBW = [ ]

   HPBW = [ ]

   SLL = [ ]

   (d) A uniform array of N elements has $N - 2$ sidelobes in one period of ψ. Count the sidelobes that fall in the visible region for this array, and explain the count using your answer to part (a).

3. **Pattern multiplication.** Four short dipoles lie end to end along the array axis, spaced $d = \lambda/2$ and fed in phase. In scan angle the short dipole's element factor is $EF(\theta) = \cos(\theta)$. The four-element uniform array factor has its first sidelobe at $\theta = 47.1^\circ$, where $AF_4 = -11.3$ dB .

   (a) Evaluate the element factor at the sidelobe angle in dB, and use it to estimate the first sidelobe level of the total pattern.

   EF at 47.1° = [ ]

   Total SLL $\approx$ [ ]

   (b) The array factor of this array has nulls at $\pm 30^\circ$ and $\pm 90^\circ$. Which nulls survive in the total pattern, and what is the total pattern at $\theta = 90^\circ$?

   $|F(90^\circ)|$ = [ ]

   (c) The element factor changed the sidelobe by 3.3 dB but changed the half-power beamwidth by almost nothing. Explain why, in terms of how $\cos(\theta)$ varies near broadside.

   (d) A colleague proposes fixing a beam that is too wide by replacing the short dipoles with more directive elements, keeping N and d the same. Say whether that works and what it would change instead.

4. **Spacing, the visible region, and grating lobes.** A uniform linear array is steered to θ_0 by a progressive phase, so

$$\psi = kd(\sin(\theta) - \sin(\theta_0))$$

and grating lobes appear wherever

$$\sin(\theta_g) = \sin(\theta_0) \pm \frac{m\lambda}{d}$$

(a) An unsteered array uses $d = \lambda$. Find every grating lobe in the visible region, and give the width of the visible region in ψ.

θ_g =

ψ width =

(b) A second array uses $d = 0.75\lambda$ and is steered to $\theta_0 = 30^\circ$. Apply the avoidance criterion, then find any grating lobe that is present.

d_{max} =

θ_g =

(c) You must scan to $\pm 45^\circ$ at 10.3 GHz with no grating lobe. Find the largest usable element spacing in millimeters, and state the trade-off that stops you from simply making d much smaller.

d_{max} =

(d) In the lab you will turn on every third element of the PHASER, making the effective spacing 42 mm at 10.3 GHz. Predict the grating lobe angles for the unsteered array.

$\theta_g =$

5. **The array as a sampled aperture.** Two unsteered uniform arrays are candidates for the same job. Array A has $N = 8$ elements at $d = \lambda/2$. Array B has $N = 4$ elements at $d = \lambda$.

   (a) Find the total length and the half-power beamwidth of each array.

   Nd (both) =

   HPBW (both) =

   (b) Find the first sidelobe level and the grating lobe angles of each array.

   A: θ_g =

   B: θ_g =

   (c) Array B uses half the elements for the same beamwidth. State which array you would build and why, and say what happens to Array B's pattern the moment we steer it off broadside.

Documentation: _____