Learning Objective 2.1: I can describe the radiation behavior of simple resonant antennas (isotropic radiator, half-wave dipole, monopole, loop) and calculate their gain and impedance.

Show your work. A **Documentation** line at the end is required (write **None** if you did not collaborate).

1. **The reference that cannot be built.** Every gain figure you will ever quote is a ratio against an antenna nobody has ever made.

   (a) Explain, in one or two sentences, why an isotropic radiator cannot exist. Your answer must mention what a real antenna is forced to have that an isotropic radiator does not.

   (b) A transmitter delivers 5W to a lossless half-wave dipole. Find the EIRP in watts and in dBm .

   EIRP =

   (c) A vendor lists an antenna at 8 dB over a dipole. What transmit power is needed to reach an EIRP of 40 dBm ?

   P_t =

   (d) Why do spectrum regulators write emission limits in EIRP rather than in transmitter power? Answer in one or two sentences.

2. **Cut a dipole for the 915 MHz ISM band.** You are building a resonant half-wave dipole for a 915MHz telemetry link out of thin copper wire.

   (a) Find the free-space wavelength, the resonant total length using the 5% rule, and the length of each arm.

   L =

   arm =

   (b) Compute $2D^2/\lambda$ for this antenna. Then state how far away you would actually stand to measure its pattern, and why that is not the same number.

   $2D^2/\lambda$ =

   use $r >$

   (c) The radio delivers 100mW to this dipole. Find the EIRP in dBm .

   EIRP =

   (d) You build it and the analyzer shows resonance at 880MHz, not 915MHz. Which way must you trim, and by how much per arm? Justify the direction in words.

   trim

3. **Reading the half-wave pattern.** The normalized field pattern of a half-wave dipole lying along z is

$$|F(\theta)| = \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}$$

(a) Evaluate the pattern at $\theta = 45^\circ$ and express it in dB relative to the broadside peak.

$|F|$ =

(b) Show that $\theta = 51^\circ$ is a half-power point, and use it to state the HPBW.

θ_{HP} =

(c) The half-wave dipole has $D = 1.64$. Find its directivity in the direction $\theta = 45^\circ$, in dBi , and comment on the sign.

$D(45^\circ)$ =

(d) The polar plot of a half-wave dipole looks strongly directional, yet its directivity is only 1.64 (2.15 dBi). Explain the apparent contradiction in one or two sentences.

4. **Impedance, resonance, and the match.** A thin dipole cut to exactly $\lambda/2$ presents an input impedance of $73 + j42.5\Omega$; trimmed to resonance it presents about $70 + j0\Omega$ at its terminals.

   (a) Find $|\Gamma|$ and the VSWR for the untrimmed dipole on a 50Ω line.

   VSWR = [ ]

   (b) Repeat for the trimmed, resonant dipole on a 50Ω line, and again on a 75Ω line.

   VSWR_{50} = [ ]

   VSWR_{75} = [ ]

   (c) What percentage of the incident power is reflected in the untrimmed 50Ω case, and in the trimmed 50Ω case?

   $|\Gamma|^2$ = [ ]

   (d) Using your answers above, argue in one or two sentences whether an engineer with a 50Ω radio should spend effort trimming the dipole to resonance or sourcing 75Ω cable. Support the recommendation with the numbers.

5. **Why the wire is never exactly half a wavelength.** These parts are answered in words and with rules of thumb, not with long derivations.

   (a) Explain physically why a resonant dipole is cut shorter than $\lambda/2$. Name the two mechanisms.

   (b) A dipole built from thick aluminum tubing is measured to resonate at 0.46λ, shorter than the 0.475λ rule of thumb predicts. Is the measurement suspicious? Explain.

   (c) A dipole one full wavelength long has a directivity of 3.82 dBi , well above the half-wave dipole's 2.15 dBi . Give the reason engineers do not simply use the longer one everywhere.

   (d) At roughly what length does the broadside directivity of a straight dipole peak, what value does it reach, and what happens to the pattern beyond that length? State the design consequence in one sentence.

6. **Using the machinery.** The radiation integral gave you a pattern that works for *any* dipole length,

$$|F(\theta)| \propto \frac{\cos\left(\frac{kL}{2}\cos\theta\right) - \cos\left(\frac{kL}{2}\right)}{\sin\theta}$$

and the impedance came from two tabulated special functions, $C_{in}(2\pi) = 2.4376$ and $Si(2\pi) = 1.4182$. Use them.

(a) For a **full-wave** dipole ($L = \lambda$), evaluate the pattern at $\theta = 90^\circ$ and $\theta = 60^\circ$, then express the 60° value in dB relative to the peak.

peak at ____

60° value = ____

(b) Compute the reactance of a half-wave dipole from $X = \frac{\eta_0}{4\pi} Si(2\pi)$, and state in one sentence why the wire radius does not appear in the answer.

X = ____

(c) The resistance came out of an integral of the far-field pattern over a sphere, but the reactance did not and could not. Explain why in one or two sentences.

Documentation: _____