Learning Objective 2.1: I can describe the radiation behavior of simple resonant antennas (isotropic radiator, half-wave dipole, monopole, loop) and calculate their gain and impedance.

Show your work. A **Documentation** line at the end is required (write **None** if you did not collaborate).

1. **The reference that cannot be built.** Every gain figure you will ever quote is a ratio against an antenna nobody has ever made.

   (a) Explain, in one or two sentences, why an isotropic radiator cannot exist. Your answer must mention what a real antenna is forced to have that an isotropic radiator does not.

   In the far field the electric field is transverse, so it lies tangent to any sphere surrounding the antenna. A nonvanishing tangential vector field cannot be defined everywhere on a sphere (the "hairy ball" result), so the field must go to zero somewhere. That zero is a **null**. Every real antenna therefore has at least one null; an isotropic radiator by definition has none, so it is a unit of comparison rather than a buildable antenna.

   (b) A transmitter delivers 5W to a lossless half-wave dipole. Find the EIRP in watts and in dBm.

   The half-wave dipole has $D = 1.64$, and with negligible conductor loss $G = D$.

   $$\text{EIRP} = P_t G_t = (5\text{W})\,(1.64) = 8.2\text{W}$$
   $$P_t\,[\,\text{dBm}\,] = 10\log_{10}(5000) = 37.0\ \text{dBm}$$
   $$\text{EIRP} = 37.0\ \text{dBm} + 2.15\ \text{dBi} = 39.1\ \text{dBm}$$

   EIRP = $\boxed{8.2\text{W} = 39.1\ \text{dBm}}$

   (c) A vendor lists an antenna at 8 dB over a dipole. What transmit power is needed to reach an EIRP of 40 dBm?

   Convert the gain to the isotropic reference first:

   $$G = 8\ \text{dBd} + 2.15 = 10.15\ \text{dBi}$$
   $$P_t = \text{EIRP} - G = 40\ \text{dBm} - 10.15\ \text{dBi} = 29.9\ \text{dBm}$$
   $$P_t = 10^{29.85/10}\text{mW} = 966\text{mW} \approx 0.97\text{W}$$

   P_t = $\boxed{29.9\ \text{dBm} = 0.97\text{W}}$

   (d) Why do spectrum regulators write emission limits in EIRP rather than in transmitter power? Answer in one or two sentences.

   Interference depends on the power density arriving at a victim receiver, which is set by the transmitter power *and* the antenna gain together. EIRP combines the two into the single quantity that actually determines the field in a given direction, so a limit on EIRP cannot be evaded by keeping the transmitter small and bolting on a very high-gain antenna.

2. **Cut a dipole for the 915 MHz ISM band.** You are building a resonant half-wave dipole for a 915MHz telemetry link out of thin copper wire.

   (a) Find the free-space wavelength, the resonant total length using the 5% rule, and the length of each arm.

   $$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{915 \times 10^6} = 0.328\text{m} = 32.8\text{cm}$$
   $$L = 0.475\lambda = 0.475\,(32.8\text{cm}) = 15.6\text{cm}$$
   $$\frac{L}{2} = 7.8\text{cm per arm}$$

   Cross-check with the field form: $143/915 = 0.156$m, the same answer.

   L = 15.6cm

   arm = 7.8cm

   (b) Compute $2D^2/\lambda$ for this antenna. Then state how far away you would actually stand to measure its pattern, and why that is not the same number.

   $$\frac{2D^2}{\lambda} = \frac{2\,(0.156\text{m})^2}{0.328\text{m}} = 0.148\text{m} = 14.8\text{cm}$$

   That is less than half a wavelength, so the far-field criterion from L5 is not the binding one here. For an electrically small antenna the reactive near field is the limitation, and the practical rule is $r \gg \lambda$ — stand at several wavelengths, say $5\lambda \approx 1.6$m or more. The $2D^2/\lambda$ criterion only takes over once the antenna is large compared with a wavelength.

   $2D^2/\lambda$ = 14.8cm

   use $r >$ ≈ 1.6m

   (c) The radio delivers 100mW to this dipole. Find the EIRP in dBm.

   $$P_t = 10\log_{10}(100) = 20.0\text{ dBm}$$
   $$\text{EIRP} = 20.0\text{ dBm} + 2.15\text{ dBi} = 22.2\text{ dBm} = 164\text{mW}$$

   EIRP = 22.2 dBm

   (d) You build it and the analyzer shows resonance at 880MHz, not 915MHz. Which way must you trim, and by how much per arm? Justify the direction in words.

   Resonance low means the antenna is electrically **too long** — a longer wire resonates at a lower frequency — so trim it shorter. Length scales inversely with the resonant frequency:

   $$L_{\text{new}} = L_{\text{old}}\frac{f_{\text{old}}}{f_{\text{new}}} = 15.6\text{cm}\left(\frac{880}{915}\right) = 15.0\text{cm}$$
   $$\Delta L = 0.6\text{cm total}, \quad 0.3\text{cm per arm}$$

   Trim a little less than that and re-measure; you cannot put wire back.

   trim 0.3cm per arm

3. **Reading the half-wave pattern.** The normalized field pattern of a half-wave dipole lying along z is

$$|F(\theta)| = \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}$$

(a) Evaluate the pattern at $\theta = 45^\circ$ and express it in dB relative to the broadside peak.

$$\cos 45^\circ = 0.7071, \quad \frac{\pi}{2}(0.7071) = 1.1107 \text{ rad}$$
$$|F| = \frac{\cos(1.1107)}{\sin 45^\circ} = \frac{0.4441}{0.7071} = 0.628$$
$$|F|\,[\text{dB}] = 20\log_{10}(0.628) = -4.0 \text{ dB}$$

$|F|$ = $0.628 = -4.0$ dB

(b) Show that $\theta = 51^\circ$ is a half-power point, and use it to state the HPBW.

$$\cos 51^\circ = 0.6293, \quad \frac{\pi}{2}(0.6293) = 0.9886 \text{ rad}$$
$$|F| = \frac{0.5502}{0.7771} = 0.708 \approx \frac{1}{\sqrt{2}}$$
$$|F|\,[\text{dB}] = 20\log_{10}(0.708) = -3.0 \text{ dB}$$

By symmetry about broadside the other half-power point is at 129°, so $\theta_{HP} = 129^\circ - 51^\circ = 78^\circ$.

θ_{HP} = 78°

(c) The half-wave dipole has $D = 1.64$. Find its directivity in the direction $\theta = 45^\circ$, in dBi , and comment on the sign.

Directivity in a given direction is the peak value scaled by the normalized power pattern:

$$D(45^\circ) = D\,|F(45^\circ)|^2 = 1.64\,(0.628)^2 = 0.647$$
$$D(45^\circ)\,[\text{dBi}] = 10\log_{10}(0.647) = -1.9 \text{ dBi}$$

It is *negative*, meaning the dipole radiates less in that direction than an isotropic radiator would. A directivity above 1 in one direction is always paid for with a directivity below 1 somewhere else.

$D(45^\circ)$ = -1.9 dBi

(d) The polar plot of a half-wave dipole looks strongly directional, yet its directivity is only 1.64 (2.15 dBi). Explain the apparent contradiction in one or two sentences.

The plot is a single cut in a plane containing the wire. The pattern has *no* ϕ dependence at all, so the antenna radiates equally in every azimuth direction — it is a donut, not a beam. Directivity integrates over the whole sphere, and concentrating power in a 78°-wide band that wraps the full 360° of azimuth buys very little concentration overall.

4. **Impedance, resonance, and the match.** A thin dipole cut to exactly $\lambda/2$ presents an input impedance of $73 + j42.5\Omega$; trimmed to resonance it presents about $70 + j0\Omega$ at its terminals.

   (a) Find $|\Gamma|$ and the VSWR for the untrimmed dipole on a 50Ω line.

   VSWR = 2.18

$$\Gamma = \frac{Z_{\text{in}} - Z_0}{Z_{\text{in}} + Z_0} = \frac{23 + j42.5}{123 + j42.5}$$
$$|\Gamma| = \frac{\sqrt{23^2 + 42.5^2}}{\sqrt{123^2 + 42.5^2}} = \frac{48.3}{130.1} = 0.371$$
$$\text{VSWR} = \frac{1 + 0.371}{1 - 0.371} = 2.18$$

   (b) Repeat for the trimmed, resonant dipole on a 50Ω line, and again on a 75Ω line.

   VSWR_{50} = 1.40

   VSWR_{75} = 1.07

$$|\Gamma_{50}| = \left|\frac{70 - 50}{70 + 50}\right| = \frac{20}{120} = 0.167$$
$$\text{VSWR}_{50} = \frac{1 + 0.167}{1 - 0.167} = 1.40$$
$$|\Gamma_{75}| = \left|\frac{70 - 75}{70 + 75}\right| = \frac{5}{145} = 0.034$$
$$\text{VSWR}_{75} = \frac{1 + 0.034}{1 - 0.034} = 1.07$$

   (c) What percentage of the incident power is reflected in the untrimmed 50Ω case, and in the trimmed 50Ω case?

   $|\Gamma|^2$ = 13.8% vs 2.8%

$$|\Gamma|^2_{\text{untrimmed}} = (0.371)^2 = 0.138 = 13.8\%$$
$$|\Gamma|^2_{\text{trimmed}} = (0.167)^2 = 0.028 = 2.8\%$$

   (d) Using your answers above, argue in one or two sentences whether an engineer with a 50Ω radio should spend effort trimming the dipole to resonance or sourcing 75Ω cable. Support the recommendation with the numbers.

   Trimming. Removing the $+j42.5\Omega$ of reactance takes the 50Ω VSWR from 2.18 to 1.40 — from 13.8% reflected power down to 2.8% — while the remaining 70 versus 50Ω resistance mismatch is worth only about 0.4 of a VSWR point. The reactance costs far more than the resistance offset does, and it is removed for free with a pair of side cutters.

5. **Why the wire is never exactly half a wavelength.** These parts are answered in words and with rules of thumb, not with long derivations.

   (a) Explain physically why a resonant dipole is cut shorter than $\lambda/2$. Name the two mechanisms.

   End effect: capacitance from the tips of the dipole to the surroundings lets charge accumulate past where the metal stops, so the standing wave behaves as though the wire extended further than it physically does. **Wire thickness**: a fatter element has more end capacitance and a lower characteristic impedance, shortening it further. Both mechanisms slow the wave traveling on the wire relative to free space, and a slower wave needs less physical length to make up an electrical half wavelength. Practical resonance therefore lands near 0.47λ to 0.48λ.

   (b) A dipole built from thick aluminum tubing is measured to resonate at 0.46λ, shorter than the 0.475λ rule of thumb predicts. Is the measurement suspicious? Explain.

   No — it is exactly what the thickness mechanism predicts. The 5% rule assumes ordinary thin wire; a fat tubular element carries more end capacitance and shortens further, so 0.46λ is a reasonable resonant length for it. The rule of thumb is a starting point for cutting, not a specification.

   (c) A dipole one full wavelength long has a directivity of 3.82 dBi , well above the half-wave dipole's 2.15 dBi . Give the reason engineers do not simply use the longer one everywhere.

   Its feed point is the problem, not its pattern. At $L = \lambda$ the standing-wave current has a *null* at the center, so the terminals see a very small current for a given voltage and the input impedance climbs to hundreds or thousands of ohms — wildly mismatched to any ordinary line and extremely sensitive to construction details. The half-wave dipole puts the current maximum at the feed instead, which is worth far more in practice than 1.7 dB of directivity.

   (d) At roughly what length does the broadside directivity of a straight dipole peak, what value does it reach, and what happens to the pattern beyond that length? State the design consequence in one sentence.

   Broadside directivity peaks near $L \approx 1.25\lambda$ at about 5.2 dBi . Past that length the accumulated phase reversals on the wire split the main lobe, and the peaks walk off broadside entirely — by 1.5λ the strongest lobes sit near 43° and 137° and broadside gain has collapsed. **Consequence:** a straight wire cannot be made into a high-gain broadside antenna simply by making it longer; beyond about 1.25λ you must switch to an array, which is where Module 3 begins.

6. **Using the machinery.** The radiation integral gave you a pattern that works for *any* dipole length,

$$|F(\theta)| \propto \frac{\cos\left(\frac{kL}{2}\cos\theta\right) - \cos\left(\frac{kL}{2}\right)}{\sin\theta}$$

and the impedance came from two tabulated special functions, $C_{in}(2\pi) = 2.4376$ and $Si(2\pi) = 1.4182$. Use them.

(a) For a **full-wave** dipole ($L = \lambda$), evaluate the pattern at $\theta = 90°$ and $\theta = 60°$, then express the $60°$ value in dB relative to the peak.

peak at $90°$

$60°$ value = -4.8 dB

With $L = \lambda$, $kL/2 = \pi$, so $\cos(kL/2) = -1$.

$$\theta = 90°: \quad |F| = \frac{\cos(0) - (-1)}{1} = 2$$

$$\theta = 60°: \quad |F| = \frac{\cos(\pi/2) - (-1)}{\sin 60°} = \frac{1}{0.8660} = 1.155$$

$$\text{normalized} = \frac{1.155}{2} = 0.577$$

$$= 20\log_{10}(0.577) = -4.8 \text{ dB}$$

The peak is still at $90°$, so a full-wave dipole is still broadside.

(b) Compute the reactance of a half-wave dipole from $X = \frac{\eta_0}{4\pi} Si(2\pi)$, and state in one sentence why the wire radius does not appear in the answer.

X = 42.5Ω

$$\frac{\eta_0}{4\pi} = \frac{377}{4\pi} = 30.0\Omega$$

$$X = 30.0\,(1.4182) = 42.5\Omega$$

Every radius-dependent term in the induced-EMF expression is multiplied by $\sin(kL)$, and at exactly $L = \lambda/2$ we have $kL = \pi$, so $\sin(\pi) = 0$ and those terms vanish. The radius independence holds *only* at exactly a half wavelength.

(c) The resistance came out of an integral of the far-field pattern over a sphere, but the reactance did not and could not. Explain why in one or two sentences.

A far-field power integral accounts only for power that *leaves* the antenna and never returns, which is precisely the definition of the real part. Reactance represents energy stored in the reactive near field and handed back to the source every cycle; it never crosses the far-field sphere, so no amount of pattern integration can reveal it. Getting it requires a near-field calculation — the induced-EMF method.

Documentation: _____