Learning Objective 1.6: I can set up and interpret the radiation integrals to derive the far-field pattern of a current distribution.

Show your work. Write a **Documentation** line at the end (**None** if you did not collaborate).

1. **Setting up the integral.** Answer each in one or two sentences (or one equation).

   (a) Why do we solve for the magnetic vector potential $\vec{A}$ instead of attacking $\vec{E}$ directly? What property of $\vec{B}$ guarantees $\vec{A}$ exists?

   (b) The far field uses *two different* approximations to $R = |\vec{r} - \vec{r}'|$. Write both and state where you use each.

   (c) Write the far-field vector potential in the form (spherical wave) $\times$ (radiation vector), and say what each factor depends on.

   (d) A $\hat{\mathbf{z}}$-directed current gives $N_\theta = -N_z \sin(\theta)$. Where does that $\sin(\theta)$ come from, and what does it predict about radiation along the wire axis?

2. **The term we throw away.** A reflector antenna has largest dimension $D = 0.8$m and operates at 6GHz. The quadratic term dropped from the expansion of R leaves a worst-case phase error $\Delta\phi = \pi D^2/(4\lambda r)$.

   (a) Find λ.

   λ =

   (b) A test range puts the source at $r = 20$m. Find $\Delta\phi$ in degrees.

   $\Delta\phi$ =

   (c) Find the smallest r for which $\Delta\phi \leqslant 22.5^\circ$ ($\pi/8$). Compare it to $2D^2/\lambda$.

   r_{min} =

   (d) The range in part (b) is too short. Is the measured *pattern* wrong because the amplitude approximation failed, the phase approximation failed, or both?

3. **Uniform line source.** A $\hat{z}$-directed filament carries a uniform current I_0 over $-L/2 \leqslant z' \leqslant L/2$, with $L = 3\lambda$. Recall the space factor $S(\theta) = |\sin(u)/u|$ with $u = \frac{kL}{2}\cos(\theta)$.

   (a) Evaluate the radiation integral to show where $S(\theta)$ comes from.

   N_z =

   (b) Find the angles of the first nulls.

   θ_{null} =

   (c) Estimate the half-power beamwidth with $0.886\lambda/L$, then check it against the exact half-power condition $u = 1.392$.

   rule of thumb =

   exact =

   (d) The design is changed to $L = 6\lambda$. What happens to the beamwidth, and what happens to the first sidelobe level? Justify both from the Fourier relationship.

4. **Steering with a phase slope.** A uniform line source of length $L = 4\lambda$ carries a progressive phase that moves the pattern peak off broadside to $\theta_0 = 60^\circ$. The excitation is $I(z') = I_0 e^{-jkz'\cos(\theta_0)}$.

   (a) Show from the radiation integral that the peak lands at θ_0, and state the peak angle for this design.

   θ_{peak} = [ ]

   (b) Find the total phase difference (in degrees) between the two ends of the aperture.

   $\Delta\phi$ = [ ]

   (c) The steered beamwidth is approximately $0.886\lambda/(L\sin(\theta_0))$. Evaluate it and compare with the broadside value.

   HPBW = [ ]

5. **The half-wave dipole.** The sinusoidal current $I(z') = I_0 \sin\left(k\left(\frac{L}{2} - |z'|\right)\right)$ with $L = \lambda/2$ produces the far-field pattern

$$|F(\theta)| = \left| \frac{\cos\left(\frac{\pi}{2}\cos(\theta)\right)}{\sin(\theta)} \right|$$

(a) Evaluate $|F|$ at $\theta = 90^\circ$ and at $\theta = 0^\circ$, and interpret each.

$|F(90^\circ)|$ =

$|F(0^\circ)|$ =

(b) Evaluate $|F|$ at $\theta = 45^\circ$, in dB relative to the peak.

$|F(45^\circ)|$ =

(c) An infinitesimal dipole has $|F| = \sin(\theta)$, which at 45° is 0.707 (-3.0 dB). The half-wave dipole is 1 dB further down at the same angle. Explain physically why the half-wave pattern is *sharper*, in terms of the current distribution.

(d) The radiation integral is exact once $\vec{J}$ is known. State what makes $\vec{J}$ hard to know, and name the numerical method that solves for it.

6. **Reading the Fourier relationship.** Four aperture distributions of the *same* length L produce first sidelobes of -13.3 dB , -23 dB , -26.5 dB , and -31.5 dB . Work these parts from the **taper table in Lesson 6**, which pairs each distribution with its first-sidelobe level and also records the price: relative to uniform illumination, a taper broadens the main beam by roughly $1.3\times$ to $1.6\times$.

   (a) Match each level to its distribution: uniform, cosine taper, cosine-squared taper, triangular.

   (b) All four have the same L. Rank their half-power beamwidths, put an approximate broadening factor on each, and explain the trade.

   (c) Your requirement is a 2° beam with sidelobes below -25 dB . Which knob sets which requirement? Size the aperture, in wavelengths, both before and after you apply the taper you chose.

L (uniform) = ☐

L (tapered) = ☐

Documentation: _____