Learning Objective 1.5: I can identify and distinguish the reactive near-field, radiating near-field, and far-field regions and calculate the boundaries for a given antenna.

Show your work. Write a **Documentation** line at the end (**None** if you did not collaborate).

1. **Three terms in one equation.** The exact E_θ of an infinitesimal dipole is a sum of three pieces, and kr alone sets their relative sizes:

$$E_\theta \propto \underbrace{\frac{1}{r}}_{\text{radiation}} + \underbrace{\frac{1}{kr^2}}_{\text{induction}} + \underbrace{\frac{1}{k^2r^3}}_{\text{electrostatic}}$$

Throughout this question the antenna operates at $f = 915\text{MHz}$.

(a) We call the innermost region *reactive*. Justify that word in one or two sentences, using what the complex Poynting vector does there.

Close in, $\vec{E}$ and $\vec{H}$ are 90° out of phase, so the radial Poynting vector $\frac{1}{2}E_\theta H_\phi^*$ is almost purely *imaginary* — and its imaginary part falls off as $1/r^5$, which is why it overwhelms everything near the antenna. Imaginary power is not transported: energy flows outward for a quarter cycle and all the way back the next. That is stored energy, the same reactive power you met at the terminals in Lesson 4, now living in the surrounding field.

(b) Normalizing the radiation term to 1, give the relative sizes of the three terms at $kr = 0.1$ and at $kr = 10$, and name the dominant term in each case.

Dividing through by the radiation term leaves powers of $1/kr$:

$$\text{radiation} : \text{induction} : \text{electrostatic} = 1 : \frac{1}{kr} : \frac{1}{(kr)^2}$$

$$kr = 0.1 : \quad = 1 : 10 : 100$$

$$kr = 10 : \quad = 1 : 0.1 : 0.01$$

At $kr = 0.1$ the $1/r^3$ electrostatic term is $100\times$ the radiation term. At $kr = 10$ the $1/r$ radiation term is all that is left standing.

$kr = 0.1$: $1{:}10{:}100$, $1/r^3$ wins

$kr = 10$: $1{:}0.1{:}0.01$, $1/r$ wins

(c) Find the distance at which the radiation and induction terms are equal. Give it in wavelengths and in centimeters at 915MHz.

The two are equal when $1/kr = 1$:

$$kr = 1 \quad \longrightarrow \quad r = \frac{1}{k} = \frac{\lambda}{2\pi} \approx 0.159\lambda$$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{915 \times 10^6} = 0.328\text{m}$$

$$r = \frac{0.328}{2\pi} = 0.0522\text{m} = 5.22\text{ cm}$$

$r =$ $0.159\lambda = 5.22$ cm

(d) A colleague measures this antenna's gain with a probe held 2 cm away. In two sentences, say what is wrong with the number he reports.

At 2 cm he is well inside the 5.2 cm crossover, where the stored $1/r^2$ and $1/r^3$ terms dominate — most of what the probe picks up was never radiated and would have returned to the antenna anyway. Worse, the probe sits in the stored field and *loads* the antenna, shifting its input impedance and its resonance, so the reading describes a two-body system rather than the antenna. Gain only means something in the far field; there is no valid gain number to be had at 2 cm.

2. **An electrically large antenna.** An X-band ground-station reflector has diameter $D = 3.0$m and operates at $f = 8$GHz. The boundaries are $r_{\text{react}} = 0.62\sqrt{D^3/\lambda}$ and $r_{\text{ff}} = 2D^2/\lambda$.

   (a) Find λ and the electrical size D/λ. Is this antenna electrically large?

   $$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{8 \times 10^9} = 0.0375\text{m}$$
   $$\frac{D}{\lambda} = \frac{3.0}{0.0375} = 80$$

   Eighty wavelengths across — emphatically large, so the $2D^2/\lambda$ machinery is the right tool.

   $\lambda =$ | 0.0375m

   $D/\lambda =$ | 80 (large)

   (b) Compute both region boundaries.

   $$r_{\text{react}} = 0.62\sqrt{\frac{(3.0)^3}{0.0375}} = 0.62\sqrt{720}$$
   $$= 0.62(26.83) = 16.6\text{m}$$
   $$r_{\text{ff}} = \frac{2(3.0)^2}{0.0375} = \frac{18}{0.0375} = 480\text{m}$$

   $r_{\text{react}} =$ | ≈ 16.6m

   $r_{\text{ff}} =$ | 480m

   (c) The only outdoor range on base is 150m long. Which region does that put the source in, and what fraction of the far-field distance is it?

   $$16.6\text{m} < 150\text{m} < 480\text{m}$$

   so 150m lands in the **radiating near-field** (Fresnel) region: the energy is leaving the antenna, but the pattern is still a function of range.

   $$\frac{150}{480} = 0.31$$

   The range is at 31% of the distance it needs to be.

   region = | rad. near-field

   fraction = | $0.31\, r_{\text{ff}}$

   (d) You need this dish's sidelobe pattern and you cannot build a 480m range. Give two ways out and state what each costs.

   Near-field scanning: probe the field on a surface a few wavelengths from the aperture and transform the measured near field into the far-field pattern mathematically. It fits indoors and is the standard answer for large apertures — but it costs a precision positioner, careful probe correction, and hours of scan time, and it is only as good as your knowledge of the probe. **Compact range**: illuminate the dish with a shaped reflector that collimates the source into a locally plane wave over a quiet zone, buying you a flat wavefront in a 20m chamber. It costs a large, expensive, precisely figured reflector, and the quiet zone limits the size of what you can test. (A defensible third: test at a lower frequency, since r_{ff} scales as $1/\lambda$ — but then you are not measuring the antenna you ship.)

3. **An electrically small antenna.** A 915MHz ISM whip on a sensor node has largest dimension $D = 8$ cm. Recall $\lambda = 0.328$m at this frequency.

   (a) Compute D/λ, $2D^2/\lambda$, and $0.62\sqrt{D^3/\lambda}$.

$$\frac{D}{\lambda} = \frac{0.08}{0.328} = 0.244$$

$$\frac{2D^2}{\lambda} = \frac{2(0.08)^2}{0.328} = \frac{0.0128}{0.328} = 0.039\text{m} = 3.9\text{ cm}$$

$$0.62\sqrt{\frac{D^3}{\lambda}} = 0.62\sqrt{\frac{5.12\times10^{-4}}{0.328}} = 0.62(0.0395) = 2.5\text{ cm}$$

D/λ =	0.244
$2D^2/\lambda$ =	3.9 cm
$0.62\sqrt{D^3/\lambda}$ =	2.5 cm

   (b) Your $2D^2/\lambda$ came out *smaller than a wavelength*. Explain in two or three sentences why that number is not the far-field boundary here, and say what replaces it.

The $0.62\sqrt{D^3/\lambda}$ and $2D^2/\lambda$ formulas are aperture formulas: they assume the antenna is *electrically large* ($D > \lambda$), so that the spread in path length across the structure is what limits you. Here $D = 0.244\lambda$, so there is essentially no path-length spread to worry about and $2D^2/\lambda = 3.9$ cm is a statement about nothing. What limits you instead is the **reactive near field** of the element itself, which extends to about $\lambda/2\pi$ — exactly the $kr = 1$ crossover of Question 1. Beyond that, the radiation term is the only one left, so the far field effectively begins there.

   (c) Compute $\lambda/2\pi$ for this antenna, and state the distance at which you would actually stand to measure its pattern (use the $r \geqslant 5\lambda$ practical rule).

$$\frac{\lambda}{2\pi} = \frac{0.328}{2\pi} = 0.052\text{m} = 5.2\text{ cm}$$

$$r_{\text{meas}} \geqslant 5\lambda = 5(0.328) = 1.64\text{m}$$

$\lambda/2\pi$ =	5.2 cm
r_{meas} =	$\geqslant 1.64$m

The 5λ rule buys margin: at $\lambda/2\pi$ the radiation term has only just taken over, while at 5λ the stored terms are down by a further factor of 30 in field.

   (d) You are offered a 3m anechoic chamber to characterize this whip. Is it long enough? Name the error source that actually limits the measurement, given your answer to (c).

Yes — 3m is nearly 10λ, comfortably past the 1.64m you need, and the aperture criterion never binds for an antenna this small. The binding error source is no longer *distance*: it is everything else in the room. Chamber wall reflections, the positioner, and above all the common-mode current on the feed cable — which on an antenna only 0.24λ long can radiate comparably to the whip itself — set the accuracy floor. Choke the cable and check the quiet-zone reflectivity; do not spend money on a longer chamber.

4. **Why ranges are long: the $\pi/8$ criterion.** A source a distance r away is farther from the *edge* of an aperture of size D than from its center, by a path difference $\Delta \approx D^2/(8r)$. The agreed tolerance is $\Delta \leqslant \lambda/16$. Take the lesson's reflector: $D = 1.2$m at $f = 10$GHz, so $\lambda = 0.03$m.

   (a) At $r = 30$m, find the path difference Δ and the phase error it produces across the aperture, in degrees.

$$\Delta = \frac{D^2}{8r} = \frac{(1.2)^2}{8(30)} = \frac{1.44}{240} = 6.0 \text{ mm}$$

$$\Delta\phi = 2\pi\frac{\Delta}{\lambda} = 2\pi\left(\frac{0.006}{0.03}\right) = 1.257 \text{ rad}$$

$$= 72^\circ$$

Δ = 6.0 mm

$\Delta\phi$ = 72°

   (b) Impose $\Delta \leqslant \lambda/16$ and solve for the minimum range. Show that the $\pi/8$ tolerance is what produces the familiar $2D^2/\lambda$.

$$\frac{D^2}{8r} \leqslant \frac{\lambda}{16} \quad \longrightarrow \quad r \geqslant \frac{16D^2}{8\lambda} = \frac{2D^2}{\lambda}$$

$$r_{\text{ff}} = \frac{2(1.2)^2}{0.03} = 96\text{m}$$

r_{ff} = 96m

   A path difference of $\lambda/16$ is a phase error of $2\pi/16 = \pi/8 = 22.5^\circ$ – so "far field" and "no more than 22.5° of quadratic phase across the aperture" are the same statement.

   (c) Your chamber is the 30m one from part (a) and the program will not fund a 96m outdoor range. State what the 72° of residual phase error does to the measured pattern, and decide whether you would accept the 30m data for a *gain* check, for a *beamwidth* check, and for a *sidelobe* check.

   Quadratic phase across the aperture behaves like a defocus: it spoils the coherent addition at the edges, so the measured main lobe broadens slightly and loses a few tenths of a dB of peak, the nulls *fill in*, and the near-in sidelobes *rise* – the last effect being much the largest. **Gain**: marginal, and it will read low; usable only with a stated correction. **Beamwidth**: acceptable, since the main lobe is the least sensitive feature. **Sidelobes**: reject – at 72° the sidelobe structure is an artifact of the range, and a -25 dB requirement cannot be verified this way. If sidelobes are the deliverable, pay for near-field scanning.

   (d) Why is 22.5° "good enough"? Answer in one or two sentences, saying what would change if the tolerance were tightened to $\pi/16$.

   It is a deliberate engineering compromise: at 22.5° the residual pattern error is a few tenths of a dB in gain and a fraction of a dB in the first sidelobes – small compared with the other uncertainties on a real range – so nothing is gained by going further. Halving the tolerance to $\pi/16$ would *double* the required range length, because r goes inversely with the allowed phase error, and 96m would become 192m for an improvement no one could measure.

5. **Two antennas on one airframe.** A 3GHz ($\lambda = 0.1$m) installation carries a half-wave dipole ($D = \lambda/2 = 0.05$m) and a 2.0m reflector, mounted 10m apart.

   (a) Inside $2D^2/\lambda$ the field is radiating — energy is leaving and never coming back — and yet the *pattern* still changes as you walk outward. Explain why, in words.

   The pattern is a superposition: every patch of the aperture sends a contribution to your observation point, and what you measure is those contributions adding with their relative phases. Close in, the patches are at meaningfully *different* distances from you, so those relative phases depend on where you are standing — move outward and the interference re-forms into a different angular pattern. As r grows the quadratic phase across the aperture, $\propto D^2/r$, shrinks; past $2D^2/\lambda$ it is below 22.5° and the rays are parallel enough that the relative phases stop depending on r altogether. From there the pattern is frozen and only the amplitude keeps falling as $1/r$.

   (b) Compute $2D^2/\lambda$ for each antenna.

   dipole: $\dfrac{2(0.05)^2}{0.1} = \dfrac{0.005}{0.1} = 0.05$m

   dish: $\dfrac{2(2.0)^2}{0.1} = \dfrac{8}{0.1} = 80$m

   dipole = 0.05m

   dish = 80m

   A factor of 1600 between two antennas sharing a frequency — *size* sets the far-field distance, not wavelength alone.

   (c) At the 10m separation, which region is each antenna in with respect to the other?

   The dish sits at 10m from the dipole, and $10\text{m} \gg 0.05$m — it is deep in the dipole's **far field**. The dipole sits at 10m from the dish, well short of 80m. Check the inner boundary:

   $$0.62\sqrt{\frac{(2.0)^3}{0.1}} = 0.62\sqrt{80} = 5.5\text{m}$$

   Since $5.5\text{m} < 10\text{m} < 80$m, the dipole is in the dish's **radiating near-field**. The relationship is not symmetric.

   dish: dipole's far field

   dipole: dish's rad. near fld

   (d) The EMC engineer wants to predict the coupling between the two with Friis. Is that legitimate here? Say what you would do instead, and what it costs.

   No. Friis assumes each antenna sees the other as a point source radiating its far-field pattern — it needs *both* ends in the far field, and the dipole is $8\times$ too close to the dish for that. Used anyway, Friis would quote the dish's boresight gain at a range where that gain has not formed, and the answer could be wrong by many dB in either direction. Do it properly with a full-wave simulation of the installed geometry (method of moments or FDTD over the airframe), or measure the coupling directly with a network analyzer on the real structure. Both cost far more than an equation: the simulation needs an accurate airframe model and hours of solve time, and the measurement needs the actual aircraft.

Documentation: _____