Learning Objective 1.4: I can calculate input impedance, feed considerations, and the role of baluns in an antenna feed system.

Show your work. Write a **Documentation** line at the end (**None** if you did not collaborate).

1. **Radiation resistance and efficiency.** A short dipole has length $\ell = 0.08\lambda$. Its conductors contribute a loss resistance of $R_{\text{loss}} = 1.5\,\Omega$.

   (a) Compute the radiation resistance $R_{\text{rad}} = 80\pi^2(\ell/\lambda)^2$.

   $$R_{\text{rad}} = 80\pi^2(0.08)^2 = 789.6 \times 0.0064 \approx 5.05\,\Omega$$

   R_{rad} = $\approx 5.05\,\Omega$

   (b) Compute the radiation efficiency η_{rad}.

   $$\eta_{\text{rad}} = \frac{R_{\text{rad}}}{R_{\text{rad}} + R_{\text{loss}}} = \frac{5.05}{6.55} \approx 0.77$$

   η_{rad} = $\approx 0.77\,(77\%)$

   (c) The short dipole's directivity is $D = 1.5$ (1.76 dBi). Find the gain in dBi.

   $$G_{\text{dBi}} = 1.76 + 10\log_{10}(0.77) = 1.76 - 1.13 \approx 0.6\ \text{dBi}$$

   G = ≈ 0.6 dBi

2. **Feeding a half-wave dipole with $50\,\Omega$ coax.**

(a) A dipole trimmed to resonance presents $Z_{in} = 70 + j0\,\Omega$. Find Γ, VSWR, return loss, and mismatch loss on a $50\,\Omega$ line.

$$\Gamma = \frac{70-50}{70+50} = 0.167$$
$$\text{VSWR} = \frac{1.167}{0.833} \approx 1.40$$
$$\text{RL} = -20\log_{10}(0.167) \approx 15.6\text{ dB}$$
$$L_{mm} = -10\log_{10}\left(1 - 0.167^2\right) \approx 0.12\text{ dB}$$

Γ = 0.167 (unitless)

VSWR = ≈ 1.40

RL = ≈ 15.6 dB

L_{mm} = ≈ 0.12 dB

(b) An *untrimmed* half-wave dipole presents $Z_{in} = 73 + j42.5\,\Omega$. Find $|\Gamma|$ and VSWR on the same line.

$$|\Gamma| = \frac{\sqrt{23^2+42.5^2}}{\sqrt{123^2+42.5^2}} = \frac{48.3}{130.1} \approx 0.371$$
$$\text{VSWR} = \frac{1.371}{0.629} \approx 2.18$$

$|\Gamma|$ = ≈ 0.371

VSWR = ≈ 2.18

(c) Compare (a) and (b): in one sentence, what did trimming to resonance buy you, and why?

Trimming killed the $+j42.5\,\Omega$ of reactance, dropping VSWR from ≈ 2.2 to 1.4 — the reactance, not the resistance, was doing most of the damage.

(d) A third dipole on the same $50\,\Omega$ line measures VSWR $= 1.5$, and you know it is purely resistive at that frequency. What two real Z_{in} are consistent with that reading? State what a VSWR measurement alone can and cannot tell you.

Z_{in} = 75 or $33.3\,\Omega$

$$Z_{in} = 50 \cdot \frac{1.5}{1} = 75\,\Omega \quad \text{or} \quad Z_{in} = \frac{50}{1.5} = 33.3\,\Omega$$

VSWR comes from $|\Gamma|$ alone, so it reports the *magnitude* of the mismatch and nothing about its phase — it cannot tell you which side of Z_0 you are on, or how much of the mismatch is reactive. That is what a Smith chart, or a complex Γ measurement, is for.

3. **Quarter-wave transformer.** A resonant antenna presents a real input impedance $R_L = 36\,\Omega$. You want to match it to a $50\,\Omega$ feed line with a quarter-wave transformer.

   (a) What characteristic impedance Z_1 must the $\lambda/4$ section have?

   $$Z_1 = \sqrt{Z_0 R_L} = \sqrt{(50)(36)} = \sqrt{1800} \approx 42.4\,\Omega$$

   Z_1 = $\boxed{\approx 42.4\,\Omega}$

   (b) What is Γ at the design frequency?

   The transformer makes $Z_1^2/R_L = 1800/36 = 50\,\Omega = Z_0$, so $\Gamma = 0$.

   (c) In one sentence, what happens to the match off the design frequency, and why?

   The section is $\lambda/4$ only at the design frequency; off frequency its electrical length is no longer 90°, the transformation Z_1^2/R_L no longer holds, and $|\Gamma|$ climbs — the match is narrowband.

   (d) The antenna in this problem is purely resistive. Explain in one or two sentences why a quarter-wave transformer is the wrong tool the moment Z_{in} has a reactive part.

   A $\lambda/4$ section transforms R_L to Z_1^2/R_L — a *real* number into a *real* number. Hand it a complex load and the transformed impedance is still complex, so $\Gamma \neq 0$ no matter what Z_1 you pick. You must first cancel the reactance (or move to a matching topology that handles both parts at once, which is Question 4).

4. **L-match design.** A compact antenna presents $Z_{in} = 25 - j40\,\Omega$ at $f = 1.5$GHz. You must match it to a $50\,\Omega$ line with lumped elements. Work the match in two moves: *cancel the reactance*, then *transform the resistance*.

   (a) Find $|\Gamma|$ and VSWR for the raw, unmatched antenna.

$$\Gamma = \frac{(25 - j40) - 50}{(25 - j40) + 50} = \frac{-25 - j40}{75 - j40}$$

$$|\Gamma| = \frac{\sqrt{25^2 + 40^2}}{\sqrt{75^2 + 40^2}} = \frac{47.2}{85.0} \approx 0.555$$

$$\text{VSWR} = \frac{1.555}{0.445} \approx 3.49$$

$\lvert\Gamma\rvert$ =	≈ 0.555
VSWR =	≈ 3.49

   (b) **Move 1.** What series reactance cancels the load reactance, and what component value does it correspond to at 1.5GHz?

   Cancel $-j40\,\Omega$ with $+j40\,\Omega$ in series — an inductor.

$$\omega = 2\pi(1.5 \times 10^9) = 9.425 \times 10^9 \text{ rad/s}$$

$$L_1 = \frac{40}{9.425 \times 10^9} = 4.24\text{ nH}$$

   The load now looks like a plain $25\,\Omega$ resistor.

X_1 =	$+j40\,\Omega$
L_1 =	4.24 nH

   (c) **Move 2.** With $R_L = 25\,\Omega$ real, design the L-network that transforms it to $50\,\Omega$. Use $Q = \sqrt{Z_0/R_L - 1}$, series $X_s = QR_L$ on the load side and shunt $X_p = Z_0/Q$ on the line side. Give both reactances and both component values, and combine the two series inductors.

$$Q = \sqrt{\frac{50}{25} - 1} = 1$$

$$X_s = QR_L = 25\,\Omega \text{ (series } L)$$

$$L_2 = \frac{25}{9.425 \times 10^9} = 2.65\text{ nH}$$

$$X_p = \frac{Z_0}{Q} = 50\,\Omega \text{ (shunt } C)$$

$$C = \frac{1}{\omega X_p} = \frac{1}{(9.425 \times 10^9)(50)} = 2.12\text{ pF}$$

   The two series inductors merge: $L_1 + L_2 = 6.9$ nH ($+j65\,\Omega$). Check: $25 - j40 + j65 = 25 + j25$, whose admittance is $0.02 - j0.02$ S; the shunt capacitor adds $+j0.02$ S, leaving 0.02 S $= 50\,\Omega$ and $\Gamma = 0$.

L_{tot} =	6.9 nH
C =	2.12 pF

   (d) The loaded Q of this match came out at 1. Compare the bandwidth and the practicality of this L-match against the $\lambda/4$ transformer of Question 3, and say which you would build for a 1.5GHz handheld radio.

A matching network's fractional bandwidth goes roughly as $1/Q$, so $Q = 1$ is about as broadband as a two-element match gets — a $2{:}1$ impedance ratio is a gentle transformation. A larger ratio would force Q up and the bandwidth down; that is the fundamental cost of matching, not a property of this topology. Against the $\lambda/4$ transformer: the transformer needs a *real* load and a physical quarter wavelength of line (5 cm at 1.5GHz), while the L-match swallows the reactance directly and occupies two 0402 parts. **Build the L-match** in a handheld — board area and the complex load both argue for it; save the $\lambda/4$ transformer for a printed feed where the line is free anyway.

5. **Mismatch that kills the amplifier.** A power amplifier (PA) delivers a forward power of $P_{fwd} = +40$ dBm toward a transmit antenna. A shunt protection diode at the PA output fails catastrophically if it must dissipate more than $P_{max} = +30$ dBm . Model *all* of the reflected power as landing on the diode, so $P_{ref} = |\Gamma|^2 P_{fwd}$.

   (a) Convert P_{fwd} and P_{max} to watts.

$$P_{fwd} = 10^{(40-30)/10}\text{W} = 10\text{W}$$
$$P_{max} = 10^{(30-30)/10}\text{W} = 1\text{W}$$

P_{fwd} = 10W

P_{max} = 1W

   (b) Find the largest $|\Gamma|$ the system can tolerate before the diode is destroyed.

$$|\Gamma|_{max} = \sqrt{\frac{P_{max}}{P_{fwd}}} = \sqrt{0.1} \approx 0.316$$

$|\Gamma|_{max}$ = ≈ 0.316

   (c) Convert to a VSWR limit and a return-loss limit, and state the datasheet rule that follows.

$$\text{VSWR}_{max} = \frac{1.316}{0.684} \approx 1.93$$
$$\text{RL}_{min} = -10\log_{10}(0.1) = 10\text{ dB}$$

VSWR_{max} = ≈ 1.93

RL_{min} = 10 dB

   Rule: keep VSWR $\leqslant 1.9$ (return loss $\geqslant 10$ dB); reflected power must stay at least 10 dB below forward power.

   (d) In one sentence: how does a circulator or isolator between the PA and antenna change this picture?

   It routes the reflected wave into a matched dump-port load instead of back into the PA, so the diode no longer sees it — the antenna can then run at much higher VSWR (paid for by insertion loss, size, and bandwidth).

6. **Baluns.**

   (a) In one or two sentences, explain why feeding a dipole *directly* with coax (no balun) distorts the pattern.

   Coax is unbalanced, so the two arms present unequal impedances to it. The imbalance drives a common-mode current on the *outside* of the shield; that shield current radiates, so the feed line becomes part of the antenna and skews the pattern.

   (b) You must feed a folded dipole ($Z_{in} \approx 292\,\Omega$) with $75\,\Omega$ coax. Which balun would you choose, and what does it do?

   A **4:1 (half-wave) balun**: it balances the feed *and* transforms impedance 4:1, taking $292\,\Omega$ down to $\approx 73\,\Omega$ — a good match to $75\,\Omega$ coax.

   (c) You feed a straight resonant dipole ($\approx 70\,\Omega$) with $50\,\Omega$ coax. Which balun would you choose, and why is it a *different* choice than (b)?

   A **1:1 current (choke) balun**. The $70\,\Omega$ dipole already matches $50\,\Omega$ well (VSWR ≈ 1.4), so no impedance transformation is wanted — you only need to choke off the shield current. A 4:1 balun here would ruin the match.

Documentation: _____