Learning Objective 1.2: I can define and calculate fundamental antenna properties — gain, directivity, effective aperture, beamwidth, and sidelobe level — apply the reciprocity principle that links an antenna's transmit and receive behavior, and use the Friis transmission equation to predict received power in a link.

Show your work. Write a **Documentation** line at the end (**None** if you did not collaborate).

1. **Wave equation sanity check.** The 1-D wave equation from a lossless transmission line is

$$\frac{\partial^2 v}{\partial z^2} = LC\frac{\partial^2 v}{\partial t^2}.$$

(a) Show by direct substitution that $v(z,t) = f(z-ut)$ is a solution for any twice-differentiable f, and find u in terms of L and C.

(b) For a coaxial cable with $L = 250$ nH/m and $C = 100$ pF/m, compute the propagation speed u and the velocity factor u/c.

u =

u/c =

(c) In one sentence, explain what a "velocity factor of 0.66" (typical of RG-58 coax) means physically.

2. **Time and space frequencies.** A plane wave in free space is $\vec{E}(z,t) = \hat{x}E_0\cos(\omega t - kz)$.

   (a) For a signal at $f = 2.40\text{GHz}$ (Wi-Fi channel 6), compute ω, λ, and k.

   ω =

   λ =

   k =

   (b) How far does the wave crest travel in **one nanosecond**? Express your answer in wavelengths.

   (c) At a fixed point in space, how many full oscillations does the field complete in **one nanosecond**?

   (d) Sketch (by hand) $\vec{E}$ vs. z at $t = 0$ over a range of two wavelengths, then $\vec{E}$ vs. t at $z = 0$ over a range of two periods. Label the axes.

3. **Directivity from beamwidths.** A ground-based tracking antenna has half-power beamwidths $\theta_1 = 2.5^\circ$ in azimuth and $\theta_2 = 1.8^\circ$ in elevation.

   (a) Estimate the directivity in dBi using the pencil-beam approximation $D \approx 41{,}253/(\theta_1^\circ \theta_2^\circ)$.

   D =

   (b) You measure the antenna's radiation efficiency at 88% and its impedance mismatch $|\Gamma| = 0.25$. What is the realized gain at boresight, in dBi?

   G_{re} =

   (c) In one sentence, explain the difference between D, G, and G_{re} for this antenna.

4. **Effective area.**

(a) A parabolic dish for a satellite ground terminal has $G = 47$ dBi at $f = 12$GHz. Compute the effective area $A_e = G\lambda^2/(4\pi)$.

A_e =

(b) The dish is physically 2.4m in diameter. What is its aperture efficiency $\eta_{\text{ap}} = A_e/A_{\text{phys}}$? Is that reasonable?

η_{ap} =

(c) Keeping the same physical dish but re-illuminating it efficiently at $f = 30$GHz (Ka-band), what gain (dBi) would you expect if η_{ap} is unchanged?

G_{30} =

5. **Reading a real pattern.** Find a published radiation-pattern plot for a real antenna — a manufacturer datasheet, application note, or journal figure with a dB-labeled axis — and hand in the annotated plot with your readings. *Cite the source; answers are antenna-dependent, so the solutions grade method, not a specific number.*

   (a) Boresight direction (degrees off nose).

   (b) Half-power beamwidth (HPBW).

   (c) First-null beamwidth (FNBW), or state if no null is visible in range.

   (d) Peak sidelobe level (SLL), in dB below the main lobe.

   (e) Front-to-back ratio (F/B), in dB.

6. **Friis link with gain and effective area.** A UHF ground station at $f = 400$MHz transmits $P_t = 20$W into an antenna with $G_t = 8$ dBi . A cubesat at $r = 800$ km carries a receive antenna with $G_r = 3$ dBi and is on boresight for both antennas. Take $\lambda = c/f = 0.75$m.

   (a) Compute the on-axis power density S_{inc} at the cubesat.

   S_{inc} =

   (b) Compute the cubesat receive antenna's effective area $A_{e,r}$.

   $A_{e,r}$ =

   (c) Compute the received power $P_r = S_{\text{inc}} A_{e,r}$ and express the result in dBm.

   P_r =

   (d) The cubesat receiver needs -82.5 dBm at its terminals to close the link with the margin the program office demands, so this design is 6 dB short. Friis, $P_r = P_t G_t G_r \left(\frac{\lambda}{4\pi r}\right)^2$, says you can buy those 6 dB from P_t, from G_t, or from G_r. Pick one and defend it, saying what each of the three costs.

Documentation: _____