Learning Objective 1.2: I can define and calculate fundamental antenna properties — gain, directivity, effective aperture, beamwidth, and sidelobe level — apply the reciprocity principle that links an antenna's transmit and receive behavior, and use the Friis transmission equation to predict received power in a link.

Show your work. Write a **Documentation** line at the end (**None** if you did not collaborate).

1. **Wave equation sanity check.** The 1-D wave equation from a lossless transmission line is

$$\frac{\partial^2 v}{\partial z^2} = LC\frac{\partial^2 v}{\partial t^2}.$$

(a) Show by direct substitution that $v(z,t) = f(z-ut)$ is a solution for any twice-differentiable f, and find u in terms of L and C.

Let $\xi = z - ut$. By the chain rule $\frac{\partial^2 v}{\partial z^2} = f''(\xi)$ and $\frac{\partial^2 v}{\partial t^2} = u^2 f''(\xi)$. Substituting,

$$f''(\xi) = LCu^2 f''(\xi) \Longrightarrow u = \frac{1}{\sqrt{LC}}.$$

(b) For a coaxial cable with $L = 250$ nH/m and $C = 100$ pF/m, compute the propagation speed u and the velocity factor u/c.

$u =$ 2.0×10^8 m/s

$u/c =$ ≈ 0.67

$$LC = (2.5\times10^{-7})(1.0\times10^{-10}) = 2.5\times10^{-17}\ \text{s}^2/\text{m}^2$$

$$u = \frac{1}{\sqrt{LC}} = 2.0\times10^8\ \text{m/s}, \quad \frac{u}{c} = \frac{2.0}{3.0} \approx 0.67$$

(c) In one sentence, explain what a "velocity factor of 0.66" (typical of RG-58 coax) means physically.

Signals travel along the line at about 66% of the speed of light in vacuum: $u = 1/\sqrt{LC} \approx 0.66c$, set by the line's own L and C. Since $u = f\lambda$ still holds on the line, a wave at a fixed frequency has a wavelength on the line that is $0.66\times$ its free-space value.

2. **Time and space frequencies.** A plane wave in free space is $\vec{E}(z,t) = \hat{x}E_0\cos(\omega t - kz)$.

(a) For a signal at $f = 2.40$GHz (Wi-Fi channel 6), compute ω, λ, and k.

$$\omega = 2\pi f \approx 1.51 \times 10^{10} \text{ rad/s}$$
$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8}{2.40 \times 10^9} = 0.125\text{m}$$
$$k = \frac{2\pi}{\lambda} \approx 50.3 \text{ rad/m}$$

ω = 1.51×10^{10} rad/s

λ = 0.125m

k = ≈ 50.3 rad/m

(b) How far does the wave crest travel in **one nanosecond**? Express your answer in wavelengths.

$$c\Delta t = (3.00 \times 10^8)(10^{-9}) = 0.30\text{m} = \frac{0.30}{0.125} = 2.4\lambda$$

2.4λ

(c) At a fixed point in space, how many full oscillations does the field complete in **one nanosecond**?

$$f\Delta t = (2.40 \times 10^9)(10^{-9}) = 2.4 \text{ cycles}$$

2.4 cycles

(d) Sketch (by hand) $\vec{E}$ vs. z at $t = 0$ over a range of two wavelengths, then $\vec{E}$ vs. t at $z = 0$ over a range of two periods. Label the axes.

Two identical cosines: one vs. z with period $\lambda = 12.5$ cm, one vs. t with period $T = 1/f \approx 417$ ps. The answers to (b) and (c) are the *same number* – that is $\omega/k = c$ in disguise: cycles per meter of travel equals cycles per second of elapsed time, related by the speed of light.

3. **Directivity from beamwidths.** A ground-based tracking antenna has half-power beamwidths $\theta_1 = 2.5^\circ$ in azimuth and $\theta_2 = 1.8^\circ$ in elevation.

   (a) Estimate the directivity in dBi using the pencil-beam approximation $D \approx 41{,}253/(\theta_1^\circ \theta_2^\circ)$.

   D = ≈ 39.6 dBi

$$D \approx \frac{41{,}253}{(2.5)(1.8)} = \frac{41{,}253}{4.5} \approx 9170$$
$$D_{\text{dBi}} = 10\log_{10}(9170) \approx 39.6 \text{ dBi}$$

   (b) You measure the antenna's radiation efficiency at 88% and its impedance mismatch $|\Gamma| = 0.25$. What is the realized gain at boresight, in dBi?

   G_{re} = ≈ 38.8 dBi

$$G = \eta_{\text{rad}} D = 0.88(9170) \approx 8070 \rightarrow 39.1 \text{ dBi}$$
$$1 - |\Gamma|^2 = 1 - 0.0625 = 0.9375$$
$$G_{\text{re}} = 0.9375(8070) \approx 7560 \rightarrow 38.8 \text{ dBi}$$

   (c) In one sentence, explain the difference between D, G, and G_{re} for this antenna.

   D is purely geometric (pattern shape only); G folds in the ohmic and dielectric losses inside the antenna; G_{re} additionally subtracts the impedance-mismatch loss at the terminals — it is what you would actually measure with a source connected through a transmission line.

4. **Effective area.**

(a) A parabolic dish for a satellite ground terminal has $G = 47$ dBi at $f = 12$GHz. Compute the effective area $A_e = G\lambda^2/(4\pi)$.

A_e = $\approx 2.49\ \text{m}^2$

$$\lambda = \frac{c}{f} = 0.0250\text{m}, \quad G = 10^{4.7} \approx 50{,}120$$
$$A_e = \frac{G\lambda^2}{4\pi} = \frac{50{,}120(6.25 \times 10^{-4})}{4\pi} \approx 2.49\ \text{m}^2$$

(b) The dish is physically 2.4m in diameter. What is its aperture efficiency $\eta_{\text{ap}} = A_e/A_{\text{phys}}$? Is that reasonable?

η_{ap} = ≈ 0.55

$$A_{\text{phys}} = \pi(1.2)^2 \approx 4.52\ \text{m}^2$$
$$\eta_{\text{ap}} = \frac{2.49}{4.52} \approx 0.55$$

Realistic — real dishes fall in $\eta_{\text{ap}} \approx 0.5$–$0.7$ once illumination taper, spillover, feed/strut blockage, and surface roughness are counted.

(c) Keeping the same physical dish but re-illuminating it efficiently at $f = 30$GHz (Ka-band), what gain (dBi) would you expect if η_{ap} is unchanged?

G_{30} = ≈ 55 dBi

A_e fixed $\Rightarrow G = 4\pi A_e/\lambda^2 \propto f^2$. From 12 to 30 GHz is $2.5\times$ in f, i.e. $6.25\times$ in gain $= 10\log_{10}(6.25) \approx 8.0$ dB .

$$G_{30} \approx 47 + 8 = 55\ \text{dBi}$$

(Optimistic: higher f exposes surface roughness that erodes η_{ap}.)

5. **Reading a real pattern.** Find a published radiation-pattern plot for a real antenna — a manufacturer datasheet, application note, or journal figure with a dB-labeled axis — and hand in the annotated plot with your readings. *Cite the source; answers are antenna-dependent, so the solutions grade method, not a specific number.*

   (a) Boresight direction (degrees off nose).

   Marked at the true peak of the main lobe.

   (b) Half-power beamwidth (HPBW).

   Read at the -3 dB points, measured consistently on both sides.

   (c) First-null beamwidth (FNBW), or state if no null is visible in range.

   Read at the first nulls; "no null visible" is legitimate if the cut is monotonic to the plot edge.

   (d) Peak sidelobe level (SLL), in dB below the main lobe.

   The highest sidelobe peak, in dB below the main lobe.

   (e) Front-to-back ratio (F/B), in dB.

   Main lobe minus the level at 180°; "no distinct back lobe" is acceptable with justification. The habit here is reading pattern plots *consistently* — every later lesson reuses these same measurements.

6. **Friis link with gain and effective area.** A UHF ground station at $f = 400$MHz transmits $P_t = 20$W into an antenna with $G_t = 8$ dBi. A cubesat at $r = 800$ km carries a receive antenna with $G_r = 3$ dBi and is on boresight for both antennas. Take $\lambda = c/f = 0.75$m.

   (a) Compute the on-axis power density S_{inc} at the cubesat.

   $G_t = 8$ dBi $\rightarrow 6.31$.

   $$S_{\text{inc}} = \frac{P_t G_t}{4\pi r^2} = \frac{20(6.31)}{4\pi(8.0\times 10^5)^2} \approx 1.57\times 10^{-11}\ \text{W/m}^2$$

   S_{inc} = $\boxed{1.57\times 10^{-11}\ \text{W/m}^2}$

   (b) Compute the cubesat receive antenna's effective area $A_{e,r}$.

   $G_r = 3$ dBi $\rightarrow 2.00$.

   $$A_{e,r} = \frac{G_r\lambda^2}{4\pi} = \frac{2.00(0.5625)}{4\pi} \approx 0.0895\ \text{m}^2$$

   $A_{e,r}$ = $\boxed{\approx 0.0895\ \text{m}^2}$

   (c) Compute the received power $P_r = S_{\text{inc}}A_{e,r}$ and express the result in dBm.

   $$P_r = (1.57\times 10^{-11})(0.0895) \approx 1.40\times 10^{-12}\text{W}$$
   $$P_{r,\text{dBm}} = 10\log_{10}\left(\frac{1.40\times 10^{-12}}{10^{-3}}\right) \approx -88.5\ \text{dBm}$$

   P_r = $\boxed{\approx -88.5\ \text{dBm}}$

   (d) The cubesat receiver needs -82.5 dBm at its terminals to close the link with the margin the program office demands, so this design is 6 dB short. Friis, $P_r = P_t G_t G_r \left(\frac{\lambda}{4\pi r}\right)^2$, says you can buy those 6 dB from P_t, from G_t, or from G_r. Pick one and defend it, saying what each of the three costs.

   All three enter Friis as a plain multiplication, so a decibel is a decibel no matter which factor supplies it — the choice is an engineering one, not a physics one. P_t**:** 6 dB means going from 20W to 80W. That is the easiest change to make on the ground, but it costs prime power, cooling, and amplifier linearity, and on a licensed band it may run straight into an EIRP limit that more power cannot buy its way past. G_t**:** 8 dBi $\rightarrow$ 14 dBi roughly halves the ground antenna's beamwidth. Ground real estate and structure are cheap, so this is usually the right answer — the price is that the station must now track the pass accurately, and EIRP rises exactly as it would with more power. G_r**:** 6 dB more on the cubesat means a larger deployable antenna with a narrower beam, on a spacecraft whose attitude control may not be able to point it. Mass and pointing are the scarcest resources in the whole system. **Recommendation: buy it at** G_t**.** It is the cheapest decibel in the link and it sits on the end where size, power, and pointing hardware are all affordable — provided the EIRP limit leaves room.

Documentation: _____