Learning Objective 5.4: I can apply the electric field boundary conditions to calculate electric fields on both sides of a material boundary.

Show your work. Write a **Documentation** line at the end (**None** if you did not collaborate). Ignore fringing in every parallel-plate problem.

1. **Parallel plates.** Two plates of area A are a distance d apart, with a dielectric of permittivity ϵ between them. The plate at $z = d$ carries $+\rho_s$ and the plate at $z = 0$ carries $-\rho_s$.

   (a) Use the conductor boundary condition to find **E** between the plates, then integrate it to derive $C = \epsilon A/d$.

   (b) The plates measure 2 cm $\times$ 3 cm and are 0.5 mm apart, filled with a film of $\epsilon_r = 3.2$. Find C.

   C =

   (c) The capacitor is charged to 100 V. Find Q, ρ_s, and $|\mathbf{E}|$, and check that $|\mathbf{E}| = \rho_s/\epsilon$.

   Q =

   ρ_s =

   $|\mathbf{E}|$ =

2. **Coaxial capacitance.** A coaxial line has inner radius a, shield radius b, and a dielectric of permittivity ϵ.

   (a) Place $+\rho_\ell$ per meter on the inner conductor. Use Gauss's law on a cylinder of radius ρ and length ℓ to find **E** for $a < \rho < b$, then derive $C' = 2\pi\epsilon / \ln(b/a)$.

   (b) RG-59 has $a = 0.29$ mm, $b = 1.85$ mm, and solid polyethylene, $\epsilon_r = 2.25$. Find C' and the capacitance of a 15 m run.

   C' =

   C =

   (c) Module 3 shows that $L' = (\mu_0/2\pi)\ln(b/a)$. Use it to find Z_0 for RG-59, and explain why the cable is sold as $75\ \Omega$ line.

   Z_0 =

3. **Two layers.** Plates of area 0.01 m^2 hold two stacked layers: 1 mm with $\epsilon_{r1} = 2$ and 1 mm with $\epsilon_{r2} = 8$. The capacitor is charged to 500 V.

   (a) Explain why **D** is the same in both layers.

   (b) Find C, and check it as two capacitors in series.

   C =

   (c) Find the field and the voltage in each layer.

   E_1 =

   E_2 =

   (d) Both materials break down at 20 MV/m. Which layer fails first as the voltage rises, and at what total voltage?

Documentation: _____