Learning Objective 5.4: I can apply the electric field boundary conditions to calculate electric fields on both sides of a material boundary.

Show your work. Write a **Documentation** line at the end (**None** if you did not collaborate). In every problem, $\hat{\mathbf{n}}$ points from region 2 into region 1.

1. **Deriving the conditions.** Two dielectrics meet at a flat boundary that carries a surface charge density ρ_s.

   (a) Apply Gauss's law to a pillbox with faces of area ΔA, one in each region, and height $h \to 0$. Derive $D_{n1} - D_{n2} = \rho_s$.

   (b) With $\rho_s = 0$, let θ_1 and θ_2 be the angles between the field and the normal on each side. Show that $\tan\theta_1 / \tan\theta_2 = \epsilon_1/\epsilon_2$.

2. **Across a dielectric boundary.** The plane $z = 0$ separates region 1, $z > 0$, with $\epsilon_{r1} = 2$, from region 2, $z < 0$, with $\epsilon_{r2} = 5$. The boundary carries no free charge. Just above it, $\mathbf{E}_1 = \hat{\mathbf{x}}2 - \hat{\mathbf{y}}3 + \hat{\mathbf{z}}5$ V/m.

   (a) Find $\mathbf{E}_2$ just below the boundary.

   $\mathbf{E}_2 =$

   (b) Find $\mathbf{D}_2$.

   $\mathbf{D}_2 =$

   (c) Find the angle each field makes with the normal, and check your answers against part (b) of Question 1.

   $\theta_1 =$

   $\theta_2 =$

   (d) In which region is $|\mathbf{E}|$ larger, and why?

3. **A charged boundary.** The plane $z = 0$ separates air, region 1 at $z > 0$, from a dielectric with $\epsilon_r = 4$, region 2 at $z < 0$. The boundary carries $\rho_s = 88.5$ nC/m^2. Just above it, $\mathbf{E}_1 = \hat{\mathbf{x}}10 + \hat{\mathbf{z}}30$ kV/m.

   (a) Find $\mathbf{E}_2$.

   $\mathbf{E}_2 =$ ☐

   (b) Without the surface charge, what would E_{2z} be? Explain the difference in one sentence.

4. **At a conductor.** A conductor fills $z < 0$. Above it is a dielectric with $\epsilon_r = 2.5$, and the field just above the surface is $\mathbf{E} = \hat{\mathbf{z}}40$ kV/m.

   (a) Find the surface charge density on the conductor.

   $\rho_s =$ ☐

   (b) A classmate reports a field of $\hat{\mathbf{x}}5 + \hat{\mathbf{z}}40$ kV/m just above the same surface. Explain why the measurement cannot be right.

   (c) What is the volume charge density inside the conductor? Justify your answer with Gauss's law.

Documentation: _____