Learning Objective 5.3: I can calculate the electric field from the scalar electric potential in a variety of geometries.

Show your work. Take $\epsilon = \epsilon_0 = 8.854 \times 10^{-12}$ F/m unless told otherwise. Write a **Documentation** line at the end (**None** if you did not collaborate).

1. **Two point charges.** In free space, $Q_1 = 4$ nC sits at the origin and $Q_2 = -2$ nC sits at $(3, 0, 0)$ m.

   (a) Starting from $V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{l}$ and the field of a point charge, derive $V(r) = Q/(4\pi\epsilon r)$ with the reference $V = 0$ at infinity.

   (b) Find the potential at $P = (0, 4, 0)$ m.

   V_P = ____

   (c) Find the potential at $S = (3, 4, 0)$ m, and the work to carry a 1 nC charge from P to S.

   V_S = ____

   W = ____

   (d) A classmate says the work in part (c) depends on whether the charge is carried along a straight line or around a wide arc. Explain why it does not.

2. **The field from a potential.**

(a) Starting from $dV = -\mathbf{E} \cdot d\mathbf{l}$ and the definition of the gradient, $dV = \nabla V \cdot d\mathbf{l}$, show that $\mathbf{E} = -\nabla V$.

(b) In rectangular coordinates, $V = 20xy^2 - 5z$ V, with lengths in meters. Find **E**, and its value and magnitude at $(1, 1, 1)$ m.

$|\mathbf{E}|$ =

(c) In cylindrical coordinates, $V = 10 \cos\phi/\rho$ V. Find **E**, using $\nabla V = \hat{\boldsymbol{\rho}}\, \partial V/\partial\rho + \hat{\boldsymbol{\phi}}\, (1/\rho)\, \partial V/\partial\phi + \hat{\mathbf{z}}\, \partial V/\partial z$, and evaluate it at $\rho = 2$ m, $\phi = 0$.

E =

3. **A ring of charge.** A thin ring of radius a in the plane $z = 0$, centered on the z axis, carries a uniform line charge ρ_ℓ in free space.

   (a) Derive the potential at a point $(0, 0, z)$ on the axis.

   (b) Find **E** on the axis from $-\nabla V$.

   (c) For $a = 5$ cm and $\rho_\ell = 10$ nC/m, find V and $|\mathbf{E}|$ at $z = 5$ cm.

   V =

   $|\mathbf{E}|$ =

   (d) Show that for $z \gg a$ the potential becomes that of a point charge equal to the ring's total charge. Then explain why the result of part (a) is not enough to find the field at a point off the axis.

4. **A field that is not electrostatic.** A field is given as $\mathbf{E} = \hat{\mathbf{y}}\, x$ V/m, with x in meters.

   (a) Find $\oint \mathbf{E} \cdot d\mathbf{l}$ counterclockwise around the square with corners $(0,0)$, $(1,0)$, $(1,1)$, and $(0,1)$ m in the plane $z = 0$.

   $\oint =$ ____

   (b) Find $\nabla \times \mathbf{E}$, and state whether this field could be produced by static charges. Could it be written as $-\nabla V$?

Documentation: _____