Learning Objective 5.2: I can apply the integral and differential forms of Gauss's Law to calculate the electric field due to charge density in various geometries, including rectangular, cylindrical, and spherical coordinate systems.

Show your work. Write a **Documentation** line at the end (**None** if you did not collaborate).

1. **The point form in Cartesian coordinates.**

   (a) A small box has edges Δx, Δy, and Δz. Find the net flux of **D** out through its two faces normal to x, and show that dividing by the volume and letting the box shrink gives $\partial D_x/\partial x$.

   (b) The flux density in a region is $\mathbf{D} = \hat{\mathbf{x}}\, 2xy + \hat{\mathbf{y}}\, x^2 + \hat{\mathbf{z}}\, 3z^2$ nC/m^2, with x, y, z in meters. Find ρ_v at $(1, 2, 3)$.

   ρ_v =

   (c) $D_y = x^2$ is not zero. Explain why it adds nothing to the divergence.

2. **Cylindrical coordinates.** In the region $\rho \leqslant 2$ m, the flux density is $\mathbf{D} = \hat{\boldsymbol{\rho}}\,\rho^3/4$ nC/m^2, with ρ in meters.

   (a) Find ρ_v.

   ρ_v = ☐

   (b) Find the charge per meter of length inside $\rho = 2$ m by integrating ρ_v.

   Q/L = ☐

   (c) Check your answer with the integral form of Gauss's law on a cylinder of radius 2 m and length L.

3. **Spherical coordinates.**

(a) Show that the flux density of a point charge, $\mathbf{D} = \hat{\mathbf{r}} Q/(4\pi r^2)$, has zero divergence for $r > 0$. Then explain how Gauss's law still gives a flux of Q through a sphere around the charge.

(b) A charge distribution produces $\mathbf{D} = \hat{\mathbf{r}} k/r$ for $r > 0$, where k is a constant. Find ρ_v.

$\rho_v =$ ____

(c) Find the charge inside a sphere of radius R two ways: by integrating ρ_v, and from the flux of **D** through the sphere.

Documentation: _____