Learning Objective 5.2: I can apply the integral and differential forms of Gauss's Law to calculate the electric field due to charge density in various geometries, including rectangular, cylindrical, and spherical coordinate systems.

Show your work. Write a **Documentation** line at the end (**None** if you did not collaborate).

1. **Flux and enclosed charge.** A point charge $Q = 6$ nC sits at the center of a closed cubical surface of side 2 m.

   (a) Find the net electric flux out of the cube.

   $\Psi =$ ____

   (b) Find the flux through one face of the cube. Justify your answer from symmetry.

   $\Psi_{\text{face}} =$ ____

   (c) A second charge, -4 nC, is placed 3 m from the center, outside the cube. What is the net flux out of the cube now? Does the flux density **D** on the cube's surface change? Explain both answers.

   (d) Explain why Gauss's law, true for this cube, does not let us find **D** at a point on the cube's surface.

2. **A uniformly charged ball.** A ball of radius $a = 5$ cm in free space carries a uniform charge density $\rho_v = 2\ \mu\text{C/m}^3$.

   (a) Use symmetry to state the direction of **D** and the coordinates it depends on. Then derive **E** inside the ball, $r < a$, from Gauss's law.

   (b) Derive **E** outside the ball, $r > a$, and show that it is the field of a point charge equal to the ball's total charge.

   (c) Find E at $r = 2.5$ cm, 5 cm, and 10 cm.

$E(2.5\text{ cm}) =$ ☐

$E(5\text{ cm}) =$ ☐

$E(10\text{ cm}) =$ ☐

3. **A coaxial line.** A coaxial line has an inner conductor of radius $a = 1$ mm, a shield of radius $b = 3.5$ mm, and a PTFE dielectric with $\epsilon_r = 2.1$. The inner conductor carries $\rho_l = 2$ nC/m and the shield carries $-\rho_l$.

   (a) Derive **E** in each of the three regions $\rho < a$, $a < \rho < b$, and $\rho > b$.

   (b) Find E at $\rho = a$ and at $\rho = b$.

   $E(a)$ =

   $E(b)$ =

   (c) For safety the field anywhere in the dielectric must stay below 20 MV/m. Find the largest ρ_l the line can carry, and say where the limit is reached first.

   $\rho_{l,\text{max}}$ =

4. **Two parallel sheets.** In free space, an infinite sheet with $\rho_s = +1\ \mu\text{C/m}^2$ lies in the plane $z = 0$, and a second with $-\rho_s$ lies in the plane $z = d$.

   (a) Derive the field of a single infinite sheet from Gauss's law, using a pillbox.

   (b) Use superposition to find **E** between the sheets and outside them.

   between = ☐

   outside = ☐

   (c) This pair of sheets is the model for a parallel-plate capacitor. Explain why the field between them does not depend on d.

Documentation: _____