**Learning Objective 5.1:** I can apply Coulomb's Law to use direct integration to calculate the electric field due to charge density in various geometries, including rectangular, cylindrical, and spherical coordinate systems.

Show your work. Take $\epsilon = \epsilon_0 = 8.854 \times 10^{-12}$ F/m. Write a **Documentation** line at the end (**None** if you did not collaborate).

1. **A ring of charge.** A ring of radius $a$ in the $xy$ plane, centered on the origin, carries a uniform line charge $\rho_\ell$. From the lesson, its field at height $h$ on the axis is $\vec{E} = \hat{\mathbf{z}}\, \rho_\ell a h / 2\epsilon \left(a^2 + h^2\right)^{3/2}$.

   (a) With $a = 10$ cm and $\rho_\ell = 5$ nC/m, find the field at $h = 10$ cm.

   $E_z$ = 

   (b) The field is zero at the center of the ring and falls to zero far away. Find the height at which it is largest.

   $h$ = 

   (c) Now only the half of the ring with $0 \leqslant \phi' \leqslant \pi$ is charged. Derive the field at height $h$ on the axis.

2. **A sheet with a hole.** An annulus in the $xy$ plane, with inner radius $b$ and outer radius $a$, carries a uniform surface charge $\rho_s$.

   (a) Derive the field at height $h > 0$ on the axis.

   (b) Show that $b \to 0$ recovers the disk, and find the field of an infinite sheet with a hole of radius $b$.

   (c) With $\rho_s = 1\ \mu$C/m$^2$ and $b = 20$ cm, find the field of the infinite sheet with a hole at $h = 20$ cm, and compare it with a sheet without one.

   $E_z$ = 

3. **A thick spherical shell.** Charge fills the region $b < r < a$ with uniform density $\rho_v$; the region $r < b$ is empty.

   (a) Using the lesson's result for a thin shell, derive the field in all three regions.

   (b) With $\rho_v = 1\ \mu$C/m$^3$, $b = 1$ cm, and $a = 2$ cm, find the field at $r = 1.5$ cm and at $r = 4$ cm.

   at 1.5 cm = ☐

   at 4 cm = ☐

   (c) Check part (a): what does it give when $b \to 0$, and when $b \to a$ with the total charge held fixed?

**Documentation:** _____