Learning Objectives 4.1–4.3: I can apply the gradient, divergence, curl, and Laplacian in the three major coordinate systems; calculate surface and volume integrals in each; and apply the divergence theorem and Stokes' theorem to simplify integrals.

Show your work. Write a **Documentation** line at the end (**None** if you did not collaborate).

1. **Differential elements and integrals.**

   (a) Starting from the distance a point moves when each spherical coordinate changes by a small amount, derive the spherical volume element dv.

   (b) Find the area of the cap of a sphere of radius a that lies within $0 \leqslant \theta \leqslant \theta_0$. Check your answer at $\theta_0 = \pi$.

   $S =$ ☐

   (c) Find the volume of a spherical shell with inner radius a and outer radius b.

   $V =$ ☐

2. **Gradient, divergence, and the Laplacian.**

(a) Find the gradient of $f = xy$, and its magnitude at the point $(-4, 5, 0)$.

$|\nabla f|$ =

(b) Find the divergence of $\mathbf{F} = \hat{\boldsymbol{\rho}}\,\rho^2 + \hat{\mathbf{z}}\,z$.

$\nabla \cdot \mathbf{F}$ =

(c) Find the Laplacian of $f = e^{xyz}$.

$\nabla^2 f$ =

(d) Find the Laplacian of $f = 1/(x^2 + y^2 + z^2)$. Use the coordinate system that makes it easiest.

$\nabla^2 f$ =

3. **The divergence theorem.** Verify the divergence theorem for $\mathbf{F} = \hat{\boldsymbol{\rho}}\,\rho^2 + \hat{\mathbf{z}}\,z$ over the cylinder $0 \leqslant \rho \leqslant a$, $0 \leqslant z \leqslant h$.

   (a) Evaluate the volume integral of $\nabla \cdot \mathbf{F}$.

   (b) Evaluate the flux of $\mathbf{F}$ out of each of the three surfaces of the cylinder, and show that the total equals your answer to part (a).

4. **Curl and Stokes' theorem.** The field $\mathbf{F} = \hat{\phi}\,\rho$ circulates around the z axis.

   (a) Find $\nabla \times \mathbf{F}$.

   $\nabla \times \mathbf{F} =$ ☐

   (b) Find the circulation of **F** counterclockwise around the circle $\rho = a$ in the xy plane, directly and with Stokes' theorem.

   ☐

   (c) Find $\nabla \cdot \mathbf{F}$. In one or two sentences, explain what the divergence and the curl of this field tell you about it.

   (d) State which theorem relates a volume integral to a closed-surface integral, and which relates an open-surface integral to a closed-path integral.

Documentation: _____