Learning Objective 3.7: I can plot the transient response of a load connected to a source through a transmission line.

Show your work. Write a **Documentation** line at the end (**None** if you did not collaborate). All lines are lossless, and every source switches on at $t = 0$.

1. **Turn-on.** A 12 V step source with internal resistance $\mathsf{Z_s} = 75\ \Omega$ drives a 300 m line with $\mathsf{Z_0} = 50\ \Omega$ and $v_p = 2 \times 10^8$ m/s. The line ends in $\mathsf{Z_L} = 25\ \Omega$.

   (a) Find the one-way delay T.

   $$T = \frac{\ell}{v_p} = \frac{300}{2 \times 10^8} = 1.5\ \mu\text{s}$$

   T = $1.5\ \mu$s

   (b) Find the voltage V_1^+ launched onto the line at turn-on, and explain why the load does not affect it.

   $$V_1^+ = V_s \frac{\mathsf{Z_0}}{\mathsf{Z_s} + \mathsf{Z_0}} = 12 \times \frac{50}{125} = 4.8\ \text{V}$$

   V_1^+ = 4.8 V

   Just after turn-on only the incident wave exists, so the line looks like $\mathsf{Z_0}$ to the source. The load has no effect until the step reaches it at $t = T$.

   (c) Find Γ_L and Γ_s.

   $$\Gamma_L = \frac{25 - 50}{25 + 50} = -\frac{1}{3}$$
   $$\Gamma_s = \frac{75 - 50}{75 + 50} = 0.2$$

   Γ_L = -0.333

   Γ_s = 0.2

   (d) Find the final, steady-state voltage at the load.

   $$V_L(\infty) = V_s \frac{\mathsf{Z_L}}{\mathsf{Z_L} + \mathsf{Z_s}} = 12 \times \frac{25}{100} = 3.0\ \text{V}$$

   $V_L(\infty)$ = 3.0 V

2. **The bounce diagram.** Continue with the line in Question 1.

(a) Draw the bounce diagram through $t = 5T$, labeling each wave with its amplitude.

Each wave is the previous one times the reflection coefficient at the end it reached:

$$4.8, \quad -\tfrac{1}{3}(4.8) = -1.6, \quad 0.2(-1.6) = -0.32, \quad -\tfrac{1}{3}(-0.32) = 0.107, \quad 0.2(0.107) = 0.021 \text{ V}$$

Forward waves leave the source at 0, $2T$, and $4T$; reflected waves leave the load at T and $3T$.

(b) Find the load voltage on each interval: $T < t < 3T$, $3T < t < 5T$, and $5T < t < 7T$.

$$V_L = 4.8 - 1.6 = 3.20 \text{ V}$$
$$V_L = 3.20 - 0.32 + 0.107 = 2.99 \text{ V}$$
$$V_L = 2.99 + 0.021 - 0.007 = 3.00 \text{ V}$$

$V_L(2T)$ = 3.20 V

$V_L(4T)$ = 2.99 V

$V_L(6T)$ = 3.00 V

(c) Find the voltage at the source end of the line on $0 < t < 2T$ and $2T < t < 4T$.

$$V(-\ell) = 4.8 \text{ V}$$
$$V(-\ell) = 4.8 - 1.6 - 0.32 = 2.88 \text{ V}$$

$V(T)$ = 4.8 V

$V(3T)$ = 2.88 V

3. **The steady state.** The load voltage after many round trips is

$$V_L(\infty) = V_1^+ (1 + \Gamma_L) \sum_{n=0}^{\infty} (\Gamma_L \Gamma_s)^n$$

(a) Explain why the series converges for any pair of passive resistive terminations other than both ends being shorts or opens.

For a passive resistor, $-1 \leqslant \Gamma \leqslant 1$, with $|\Gamma| = 1$ only for a short or an open. If either end is a resistor, $|\Gamma_L \Gamma_s| < 1$, and a geometric series with ratio less than one in magnitude converges.

(b) Sum the series and show that $V_L(\infty) = V_s \dfrac{\mathsf{Z_L}}{\mathsf{Z_L} + \mathsf{Z_s}}$.

$$V_L(\infty) = \frac{V_1^+ (1 + \Gamma_L)}{1 - \Gamma_L \Gamma_s}$$

$$1 + \Gamma_L = \frac{2\mathsf{Z_L}}{\mathsf{Z_L} + \mathsf{Z_0}}, \qquad 1 - \Gamma_L \Gamma_s = \frac{2\mathsf{Z_0}\,(\mathsf{Z_L} + \mathsf{Z_s})}{(\mathsf{Z_L} + \mathsf{Z_0})\,(\mathsf{Z_s} + \mathsf{Z_0})}$$

$$V_L(\infty) = V_s \frac{\mathsf{Z_0}}{\mathsf{Z_s} + \mathsf{Z_0}} \cdot \frac{2\mathsf{Z_L}}{\mathsf{Z_L} + \mathsf{Z_0}} \cdot \frac{(\mathsf{Z_L} + \mathsf{Z_0})\,(\mathsf{Z_s} + \mathsf{Z_0})}{2\mathsf{Z_0}\,(\mathsf{Z_L} + \mathsf{Z_s})} = V_s \frac{\mathsf{Z_L}}{\mathsf{Z_L} + \mathsf{Z_s}}$$

(c) Check the result numerically with the values from Question 1.

$$\frac{4.8\,(1 - \frac{1}{3})}{1 + \frac{1}{3}(0.2)} = \frac{3.2}{1.0667} = 3.0\ \mathsf{V}$$

$V_L(\infty)$ = 3.0 V

4. **A pulse.** The worked example from the lesson ($V_s = 10$ V, $\mathrm{Z_s} = 25\,\Omega$, $\mathrm{Z_0} = 50\,\Omega$, $\mathrm{Z_L} = 150\,\Omega$, $T = 2\,\mu$s) is driven by a 10 V pulse that lasts from $t = 0$ to $t = T$ instead of a step.

   (a) Write the pulse as the sum of two steps, and explain why the load voltage is the step response minus a delayed copy of itself.

   The pulse is a 10 V step at $t = 0$ plus a -10 V step at $t = T$. The line is linear, so its response is the sum of the responses to the two steps: $V_L(t) = s(t) - s(t - T)$, where $s(t)$ is the step response.

   (b) The step response at the load is 0, then 10.00, 8.33, and 8.61 V on successive intervals of $2T$ starting at T. Find the pulse response at the load on $T < t < 2T$, $2T < t < 3T$, and $3T < t < 4T$.

   $$T < t < 2T: \quad 10.00 - 0 = 10.0 \text{ V}$$
   $$2T < t < 3T: \quad 10.00 - 10.00 = 0 \text{ V}$$
   $$3T < t < 4T: \quad 8.33 - 10.00 = -1.67 \text{ V}$$

   $V_L(1.5T) =$ **10.0 V**

   $V_L(2.5T) =$ **0 V**

   $V_L(3.5T) =$ **−1.67 V**

   (c) The load is a digital receiver. The pulse should produce one clean pulse at the load, but a second, inverted pulse arrives at $3T$. What single change to the circuit would remove it, and why?

   Make the source resistance equal to $\mathrm{Z_0} = 50\,\Omega$. The second pulse is the first reflection from the load, re-reflected by the source; with $\Gamma_s = 0$ the source absorbs it. (A $50\,\Omega$ load would also work, by removing the first reflection, but it would halve the received pulse.)

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