Learning Objective 3.6: I can use the Smith Chart to solve transmission line problems.

Show your work. Solve each part on a printed Smith chart, then check it with the stub formulas. Write a **Documentation** line at the end (**None** if you did not collaborate). All lines and stubs are lossless with $Z_0 = 50\,\Omega$ and $v_p = 2 \times 10^8$ m/s.

1. **Stub reactance.**

   (a) Find the input impedance of a shorted stub $\lambda/8$ long, and the lumped element it acts like.

   $$z_{sc} = j \tan \beta\ell = j \tan 45^\circ = j1$$
   $$Z_{sc} = +j50\,\Omega$$

   It acts like an inductor.

   Z_{sc} = $+j50\,\Omega$

   (b) Find the input impedance of an open stub $\lambda/8$ long.

   $$z_{oc} = -j\cot(\beta)\,\ell = -j\cot(4)\,5^\circ = -j1$$
   $$Z_{oc} = -j50\,\Omega$$

   It acts like a capacitor.

   Z_{oc} = $-j50\,\Omega$

   (c) Find the shortest shorted stub and the shortest open stub that add a normalized susceptance of $+j1$ in shunt.

   short: $-\cot(\beta)\,\ell = 1, \quad \beta\ell = 135^\circ, \quad \ell = 0.375\lambda$

   open: $\tan\beta\ell = 1, \quad \beta\ell = 45^\circ, \quad \ell = 0.125\lambda$

   short = 0.375λ

   open = 0.125λ

   (d) Repeat part (c) for a susceptance of $-j1$.

   short: $\cot(\beta)\,\ell = 1, \quad \ell = 0.125\lambda$

   open: $\tan\beta\ell = -1, \quad \ell = 0.375\lambda$

   short = 0.125λ

   open = 0.375λ

2. **Single-stub tuner.** The load $Z_L = 100 + j50\,\Omega$ from the Lesson 12 practice set must be matched at 500 MHz with a single shunt stub. Lesson 12 found that $d = 0.1988\lambda$ puts the admittance at $y = 1 + j1$.

   (a) Find the wavelength on the line and the distance d in centimeters.

   $$\lambda = \frac{v_p}{f} = \frac{2 \times 10^8}{5 \times 10^8} = 0.4 \text{ m}$$
   $$d = 0.1988(40 \text{ cm}) = 7.95 \text{ cm}$$

   $d =$ | 7.95 cm |

   (b) The tuner is built in coaxial cable. Choose the stub type and find its length in centimeters.

   The stub must add $-j1$. A coaxial stub should be shorted, and part 1(d) gives 0.125λ.

   $$\ell = 0.125(40 \text{ cm}) = 5.0 \text{ cm}$$

   $\ell =$ | 5.0 cm, shorted |

   (c) Find the second solution: d, y there, and the open-stub length.

   Continuing clockwise from the upper crossing at $63.4°$ to the lower one at $-63.4°$ adds $126.9°$, or 0.1762λ.

   $$d = 0.1988\lambda + 0.1762\lambda = 0.375\lambda, \qquad y = 1 - j1$$
   $$\ell_{oc} = 0.125\lambda = 5.0 \text{ cm}$$

   $d =$ | 0.375λ |

   $\ell_{oc} =$ | 0.125λ |

   (d) Explain in one or two sentences why a tuner is usually a shunt stub rather than a series stub.

   A shunt stub connects at a T junction without cutting the main line. A series stub has to be inserted into one conductor of the line, which is hard to build in coax or microstrip.

Documentation: _____