Learning Objective 3.6: I can use the Smith Chart to solve transmission line problems.

Show your work. Solve each part on a printed Smith chart, then check it with the formulas from the lesson. Write a **Documentation** line at the end (**None** if you did not collaborate). All lines are lossless with $Z_0 = 50\,\Omega$.

1. **Element moves.** State which way each element moves the point, and along which grid circle.

   (a) A series capacitor added to $z = 1 + j1$.

   It adds negative reactance, so z moves counterclockwise along the $r = 1$ circle toward the lower half of the chart.

   (b) A shunt inductor added to $y = 0.5 + j0.5$.

   It adds negative susceptance, so y moves down the $g = 0.5$ circle, counterclockwise on the same grid.

   (c) A load has $|\Gamma| = 0.5$. Find the reactance x where its VSWR circle crosses the $r = 1$ circle.

   $$x = \pm\frac{2|\Gamma|}{\sqrt{1-|\Gamma|^2}} = \pm\frac{1}{\sqrt{0.75}} = \pm 1.155$$

   $x =$ ± 1.155

2. **Series-element match.** A load $Z_L = 100 + j50\,\Omega$ must be matched at 500 MHz with a length of line and one series element.

   (a) Find Γ_L and the reactance x on the $r = 1$ circle.

   $$z_L = 2 + j1, \qquad \Gamma_L = 0.4 + j0.2 = 0.447\angle 26.6^\circ$$
   $$x = \pm\frac{2(0.447)}{\sqrt{1-0.2}} = \pm 1.0$$

   The crossings are at $\pm\cos^{-1} 0.447 = \pm 63.4^\circ$.

   $x =$ ± 1.0

   (b) Find the shortest line length d and z at that point.

   Turning clockwise from 26.6°, the first crossing is the lower one at -63.4°, a turn of 90°.

   $$d = \frac{90^\circ}{720^\circ}\lambda = 0.125\lambda, \qquad z = 1 - j1$$

   $d =$ 0.125λ

   $z =$ $1 - j1$

   (c) Name the series element and find its value.

   It must add $+j1$, or $+j50\,\Omega$: an inductor.

   $$L = \frac{50}{2\pi\,(5\times 10^8)} = 15.9\text{ nH}$$

   element = $L = 15.9$ nH

3. **Shunt-element match.** Match the same load at 500 MHz with a length of line and one shunt element.

   (a) Find y_L and the angle of its point on the chart.

   $$y_L = 0.4 - j0.2, \qquad -\Gamma_L = 0.447\angle -153.4^\circ$$

   y_L = $0.4 - j0.2$

   (b) Find the shortest d that puts y on the $g = 1$ circle, and y there.

   Turning clockwise from -153.4° passes the left real axis and reaches the upper crossing at $+63.4^\circ$ first.

   $$\text{turn} = 26.6^\circ + 116.6^\circ = 143.1^\circ$$
   $$d = \frac{143.1^\circ}{720^\circ}\lambda = 0.1988\lambda, \qquad y = 1 + j1$$

   d = 0.1988λ

   y = $1 + j1$

   (c) Name the shunt element and find its value.

   It must add $-j1$, a susceptance of $-1/50$ S: an inductor.

   $$\frac{1}{\omega L} = \frac{1}{50}, \qquad L = \frac{50}{2\pi\,(5 \times 10^8)} = 15.9 \text{ nH}$$

   element = $L = 15.9$ nH

   (d) Which of the two designs, series or shunt, would you build? Justify your answer in one or two sentences.

   The series design: its line is 0.125λ instead of 0.1988λ, so it changes less with frequency and gives the wider bandwidth. Both use the same 15.9 nH inductor, so the line length decides.

Documentation: _____