Learning Objectives 3.5 and 3.6: I can identify the various quantities represented on the Smith Chart. I can use the Smith Chart to solve transmission line problems.

Show your work. Solve each part on a printed Smith chart, then check it with the formula. Write a **Documentation** line at the end (**None** if you did not collaborate). All lines are lossless with $Z_0 = 50\,\Omega$.

1. **Input impedance by rotation.** A load $Z_L = 25 - j25\,\Omega$ terminates a line 0.125λ long.

   (a) Find z_L and Γ_L in polar form.

$$z_L = 0.5 - j0.5$$
$$\Gamma_L = \frac{-0.5 - j0.5}{1.5 - j0.5} = -0.2 - j0.4 = 0.447\angle -116.6^\circ$$

Γ_L = $0.447\angle -116.6^\circ$

   (b) Through what angle, and in which direction, does the point turn to reach the input? Find Γ_{in}.

$$2\beta d = 2\left(\frac{2\pi}{\lambda}\right)(0.125\lambda) = 90^\circ \text{ clockwise}$$
$$\Gamma_{in} = 0.447\angle(-116.6^\circ - 90^\circ) = 0.447\angle 153.4^\circ$$

Γ_{in} = $0.447\angle 153.4^\circ$

   (c) Find Z_{in}, and check it with $z_{in} = \dfrac{z_L + j\tan\beta d}{1 + jz_L\tan\beta d}$.

$$\tan\beta d = \tan 45^\circ = 1$$
$$z_{in} = \frac{0.5 + j0.5}{1.5 + j0.5} = \frac{(0.5 + j0.5)(1.5 - j0.5)}{2.5}$$
$$= 0.4 + j0.2$$
$$Z_{in} = 20 + j10\,\Omega$$

Z_{in} = $20 + j10\,\Omega$

   A capacitive load looks inductive an eighth of a wavelength away.

2. **Admittance and the quarter wave.** A load $Z_L = 100 + j50\,\Omega$.

   (a) Find y_L by reflecting Γ_L through the center, and check it with $1/z_L$.

$$z_L = 2 + j1, \qquad \Gamma_L = \frac{1+j1}{3+j1} = 0.4 + j0.2$$
$$y_L = \frac{1-\Gamma_L}{1+\Gamma_L} = \frac{0.6 - j0.2}{1.4 + j0.2} = 0.4 - j0.2$$
$$\frac{1}{z_L} = \frac{2-j1}{5} = 0.4 - j0.2$$

y_L = $0.4 - j0.2$

   (b) The load sits at the end of a quarter-wavelength line. Find Z_{in}.

$$z_{in} = \frac{1}{z_L} = 0.4 - j0.2, \qquad Z_{in} = 20 - j10\,\Omega$$

Z_{in} = $20 - j10\,\Omega$

3. **The wavelength scale.** A load has $\Gamma_L = 0.6\angle 60°$.

   (a) Find the load's reading on the wavelengths-toward-generator scale.

$$\frac{180° - 60°}{720°}\lambda = 0.1667\lambda$$

WTG = 0.1667λ

   (b) A point on the line reads 0.3λ on the same scale. How far is it from the load, and what is the angle of Γ there?

$$d = 0.3\lambda - 0.1667\lambda = 0.1333\lambda$$
$$\theta = 60° - 720°(0.1333) = -36°$$

d = 0.1333λ

θ = $-36°$

4. **Voltage maxima and minima.** The load of Question 2, $z_L = 2 + j1$, has $\Gamma_L = 0.447\angle 26.6°$.

   (a) Find the distance from the load to the first voltage maximum, and z there.

   The first maximum is where the point first reaches the right real axis, after a clockwise turn of $26.6°$.

   $$d_{\text{max}} = \frac{26.6°}{720°}\lambda = 0.0369\lambda$$
   $$z = \text{VSWR} = \frac{1.447}{0.553} = 2.62$$

   d_{max} = 0.0369λ

   z = 2.62

   (b) Find the distance to the first voltage minimum, and z there.

   $$d_{\text{min}} = 0.0369\lambda + 0.25\lambda = 0.2869\lambda$$
   $$z = \frac{1}{\text{VSWR}} = 0.382$$

   d_{min} = 0.2869λ

   z = 0.382

   (c) A measurement finds a voltage minimum. Explain in one or two sentences why the impedance there is always real and less than one.

   At a minimum, $\Gamma(d)$ points along the negative real axis, so $z = (1 - |\Gamma|)/(1 + |\Gamma|)$, which is real and less than one for any $|\Gamma| > 0$.

Documentation: _____