Learning Objective 3.5: I can identify the various quantities represented on the Smith Chart.

Show your work. Check each calculation on a printed Smith chart. Write a **Documentation** line at the end (**None** if you did not collaborate). All lines are lossless with $Z_0 = 50\,\Omega$.

1. **From impedance to Γ.** Each load below terminates a $50\,\Omega$ line.

   (a) $Z_L = 100 - j50\,\Omega$. Find z_L, Γ in rectangular and polar form, and the VSWR.

$$z_L = 2 - j1$$
$$\Gamma = \frac{1-j1}{3-j1} = \frac{(1-j1)(3+j1)}{10} = 0.4 - j0.2$$
$$= 0.447\angle -26.6^\circ$$
$$\text{VSWR} = \frac{1.447}{0.553} = 2.62$$

   Γ = $0.447\angle -26.6^\circ$

   VSWR = 2.62

   The load is in the lower half of the chart because it is capacitive.

   (b) $Z_L = 25 + j25\,\Omega$. Find Γ in polar form.

$$z_L = 0.5 + j0.5$$
$$\Gamma = \frac{-0.5+j0.5}{1.5+j0.5} = \frac{(-0.5+j0.5)(1.5-j0.5)}{2.5}$$
$$= -0.2 + j0.4 = 0.447\angle 116.6^\circ$$

   Γ = $0.447\angle 116.6^\circ$

   (c) The loads in parts (a) and (b) have the same $|\Gamma|$. What does that mean on the chart, and what does it say about their VSWRs?

   Both lie on the same circle centered on the chart, so they have the same VSWR, 2.62, even though their impedances are very different.

2. **From Γ to impedance.** Use $z = (1+\Gamma)/(1-\Gamma)$.

   (a) $\Gamma = 0.5\angle 90°$. Find Z_L.

$$z_L = \frac{1+j0.5}{1-j0.5} = \frac{(1+j0.5)^2}{1.25} = \frac{0.75+j1}{1.25}$$
$$= 0.6 + j0.8$$
$$Z_L = 30 + j40\,\Omega$$

$Z_L =$ $30 + j40\,\Omega$

   (b) $\Gamma = -0.6$. Find Z_L, and say where the point is on the chart.

$$z_L = \frac{0.4}{1.6} = 0.25, \qquad Z_L = 12.5\,\Omega$$

$Z_L =$ $12.5\,\Omega$

On the left half of the real axis, between the short and the center.

   (c) Without computing, name the load at each point: $\Gamma = -1$, $\Gamma = 0$, $\Gamma = +1$, and $\Gamma = +j$.

A short circuit, a matched $50\,\Omega$ load, an open circuit, and a pure reactance: $z = (1+j)/(1-j) = j$, an inductor of $+j50\,\Omega$.

3. **The grid circles.**

(a) Find the center and radius of the $r = 3$ circle.

$$\left(\frac{r}{1+r}, 0\right) = (0.75, 0), \qquad \frac{1}{1+r} = 0.25$$

It touches the real axis at $\Gamma = 0.5$ and $\Gamma = 1$.

center = $(0.75, 0)$

radius = 0.25

(b) Find the center and radius of the $x = 0.5$ circle.

$$\left(1, \frac{1}{x}\right) = (1, 2), \qquad \frac{1}{|x|} = 2$$

center = $(1, 2)$

radius = 2

(c) A load has a VSWR of 3. Find $|\Gamma|$, and the two resistances where its VSWR circle crosses the real axis.

$$|\Gamma| = \frac{\text{VSWR} - 1}{\text{VSWR} + 1} = \frac{2}{4} = 0.5$$

$$r_{\text{right}} = 3, \qquad r_{\text{left}} = \frac{1 - 0.5}{1 + 0.5} = \frac{1}{3}$$

$|\Gamma|$ = 0.5

crossings = $r = 3,\ 1/3$

Documentation: _____