Learning Objective 3.4: I can use the transmission line equation to match a load to a source using a length of transmission line.

Show your work. Write a **Documentation** line at the end (**None** if you did not collaborate). All lines are lossless, and d is the distance from the load toward the source.

1. **Admittance.**

   (a) Substitute $z = 1/y$ into $\Gamma = \frac{z-1}{z+1}$ and show that $\Gamma = \frac{1-y}{1+y}$.

   $$\Gamma = \frac{1/y - 1}{1/y + 1} = \frac{1 - y}{1 + y}$$

   after multiplying the numerator and denominator by y.

   (b) Starting from the input impedance of a shorted stub and of an open stub of length ℓ_s, find the normalized admittance of each.

   $$\text{shorted:} \quad y = \frac{Z_0}{jZ_0 \tan \beta\ell_s} = -j\cot(\beta)\,\ell_s$$
   $$\text{open:} \quad y = \frac{Z_0}{-jZ_0 \cot(\beta)\,\ell_s} = j\tan\beta\ell_s$$

   (c) Explain in one sentence why a matching component that connects across the line is easier to design with admittance than with impedance.

   Admittances in parallel add, so the component's admittance simply adds to the line's admittance at that point.

2. **A parallel match.** The $100\ \Omega$ load from the Lesson 8 practice set terminates a $50\ \Omega$ line at 2 GHz, with $\lambda = 10$ cm in the line. Its reflection coefficient is $\Gamma = 0.333\angle 0°$.

   (a) Find y_L and the two values of b at the matching points.

$$y_L = \frac{50}{100} = 0.5$$
$$b = \pm\frac{2 \times 0.333}{\sqrt{1 - 0.111}} = \pm 0.707$$

y_L = 0.5

b = ± 0.707

   (b) Find the phase θ_d and the distance d for each matching point.

$\tan^{-1}(0.707/2) = 19.5°$, so $\theta_d = \mp(90° + 19.5°) = \mp 109.5°$.

$$b = +0.707: \quad d = \frac{0° + 109.5°}{720°}\lambda = 0.152\lambda = 1.52 \text{ cm}$$
$$b = -0.707: \quad d = \frac{0° - 109.5° + 360°}{720°}\lambda = 0.348\lambda = 3.48 \text{ cm}$$

d_1 = 0.152λ

d_2 = 0.348λ

   (c) Find the parallel component that completes each match, and its value.

The susceptance to cancel is $0.707/50 = 14.1$ mS. At d_1, $y = 1 + j0.707$, so we add a parallel inductor:

$$L = \frac{1}{2\pi \times 2 \times 10^9 \times 0.0141} = 5.63 \text{ nH}$$

At d_2, $y = 1 - j0.707$, so we add a parallel capacitor:

$$C = \frac{0.0141}{2\pi \times 2 \times 10^9} = 1.13 \text{ pF}$$

at d_1: 5.63 nH

at d_2: 1.13 pF

3. **Stubs in place of components.** Replace each component from Question 2 with a stub of the same $50\,\Omega$ line.

(a) At d_1 the stub must supply $b_s = -0.707$. Find the shortest shorted stub and the shortest open stub that do.

shorted: $\cot(\beta)\,\ell_s = 0.707$

$\beta\ell_s = 54.7^\circ, \quad \ell_s = 0.152\lambda$

open: $\tan\beta\ell_s = -0.707$

$\beta\ell_s = 144.7^\circ, \quad \ell_s = 0.402\lambda$

shorted = 0.152λ

open = 0.402λ

(b) At d_2 the stub must supply $b_s = +0.707$. Find the shortest shorted stub and the shortest open stub that do.

shorted: $\cot(\beta)\,\ell_s = -0.707$

$\beta\ell_s = 125.3^\circ, \quad \ell_s = 0.348\lambda$

open: $\tan\beta\ell_s = 0.707$

$\beta\ell_s = 35.3^\circ, \quad \ell_s = 0.098\lambda$

shorted = 0.348λ

open = 0.098λ

(c) Which line-and-stub design uses the least total line? Give its two lengths in centimeters.

The line of 0.152λ with a shorted stub of 0.152λ, a total of 0.304λ: 1.52 cm of line and a 1.52 cm stub. The other short choice, 0.348λ with an open stub of 0.098λ, totals 0.446λ.

4. **A match at the load.** A load $Z_L = 25 - j25\,\Omega$ terminates a $50\,\Omega$ line at 2 GHz.

   (a) Find y_L.

$$y_L = \frac{1}{0.5 - j0.5} = \frac{0.5 + j0.5}{0.5} = 1 + j1$$

y_L = $1 + j1$

   (b) The conductance is already 1, so $d = 0$ is a matching point. Find the single parallel component, connected across the load, that matches it.

   It must supply $-j1$, or $-j20$ mS, so it is an inductor:

$$L = \frac{1}{2\pi \times 2 \times 10^9 \times 0.02} = 3.98\text{ nH}$$

L = 3.98 nH

Documentation: _____