Learning Objective 3.4: I can use the transmission line equation to match a load to a source using a length of transmission line.

Show your work. Write a **Documentation** line at the end (**None** if you did not collaborate). All lines are lossless, and d is the distance from the load toward the source.

1. **The matching point.** At a point on the line, the normalized impedance is $z = 1 + jx$.

   (a) Starting from $\Gamma = \frac{z-1}{z+1}$, show that $z = \frac{1+\Gamma}{1-\Gamma}$.

   (b) Show that at the point where $z = 1 + jx$,

   $$x = \pm \frac{2|\Gamma|}{\sqrt{1 - |\Gamma|^2}}.$$

   (c) Explain in one sentence why $|\Gamma|$ in part (b) is the same as the magnitude of the load's reflection coefficient.

2. **A series match.** A $100\ \Omega$ load terminates a $50\ \Omega$ line at 2 GHz. Waves travel at 2×10^8 m/s in the line, so $\lambda = 10$ cm.

   (a) Find z_L, Γ, and the VSWR.

   Γ = ____

   VSWR = ____

   (b) Find the two values of x at the matching points, and the phase θ_d of Γ at each.

   x = ____

   θ_d = ____

   (c) Find the distance d to each matching point, in wavelengths and in centimeters.

   d_1 = ____

   d_2 = ____

   (d) Find the series component that completes each match, and its value.

   at d_1: ____

   at d_2: ____

   (e) Which design would you build? Justify your choice in one or two sentences.

3. **Checking a match.** Use the transmission line equation to check the first design from Question 2.

   (a) Find Z_{in} of the $100\,\Omega$ load seen through 0.098λ of $50\,\Omega$ line.
   ($\beta d = 35.3^\circ$ and tan $\beta d = 0.707$.)

   Z_{in} =

   (b) Add the 2.81 nH series inductor. What impedance does the source side see, and what is Γ there?

   Z =

Documentation: _____