Learning Objectives 3.3 and 3.4: I can use the transmission line equation to calculate the voltage/current at an arbitrary point on a specified transmission line. I can use the transmission line equation to match a load to a source using a length of transmission line.

Show your work. Write a **Documentation** line at the end (**None** if you did not collaborate). All lines are lossless. The load is at $z = 0$ and d is the distance from the load toward the source.

1. **Deriving the input impedance.** The phasor voltage and current on a line terminated in Z_L are

$$\tilde{V}(z) = V_0^+ \left(e^{-j\beta z} + \Gamma e^{j\beta z}\right), \qquad \tilde{I}(z) = \frac{V_0^+}{\mathsf{Z}_0}\left(e^{-j\beta z} - \Gamma e^{j\beta z}\right)$$

   (a) Write $\mathsf{Z}_\mathsf{in}(d) = \tilde{V}(-d)/\tilde{I}(-d)$, substitute $\Gamma = \frac{\mathsf{Z}_\mathsf{L}-\mathsf{Z}_0}{\mathsf{Z}_\mathsf{L}+\mathsf{Z}_0}$, and show that

$$\mathsf{Z}_\mathsf{in}(d) = \mathsf{Z}_0 \frac{\mathsf{Z}_\mathsf{L} + j\mathsf{Z}_0 \tan\beta d}{\mathsf{Z}_0 + j\mathsf{Z}_\mathsf{L} \tan\beta d}.$$

   (b) Set $\mathsf{Z}_\mathsf{L} = 0$ and then $\mathsf{Z}_\mathsf{L} \to \infty$ to find the input impedance of a shorted line and of an open line.

2. **One load, three lengths.** A $100\ \Omega$ resistor terminates a $50\ \Omega$ line.

   (a) Find Z_{in} for a line $\lambda/8$ long.

   Z_{in} =

   (b) Find Z_{in} for lines $\lambda/4$ and $\lambda/2$ long.

   $\lambda/4$:

   $\lambda/2$:

   (c) The resistor absorbs real power, but the input impedance in part (a) has a reactive part. Explain in one or two sentences how both can be true.

3. **Stubs.** A circuit at 2 GHz needs a 5 nH inductor and a 1 pF capacitor. Build each from $50\,\Omega$ line in which waves travel at 2×10^8 m/s.

   (a) Find the wavelength in the line.

   $\lambda =$ ____

   (b) Find the length of the shortest shorted stub that acts as the inductor.

   $\ell =$ ____

   (c) Find the length of the shortest open stub that acts as the capacitor.

   $\ell =$ ____

   (d) Your stub in part (b) is accidentally cut 2.5 cm longer than designed. What does it act like now? Explain without computing.

4. **Quarter-wave transformer.** A $200\ \Omega$ antenna is fed at 500 MHz from a $50\ \Omega$ line. Waves travel at 2×10^8 m/s in every line.

   (a) Find the characteristic impedance and the length of the quarter-wave section that matches the antenna.

   $Z_1 =$ ____

   length = ____

   (b) The same network is used at 1 GHz. Find the impedance the $50\ \Omega$ line sees and the reflection coefficient there.

   $Z_{in} =$ ____

   $\Gamma =$ ____

5. **Power delivered.** An incident wave of amplitude $|V_0^+| = 10$ V travels on a $50\,\Omega$ line toward a $100\,\Omega$ load.

   (a) Find the incident power.

   P_{inc} =

   (b) Find the power the load absorbs.

   P_L =

   (c) Check part (b) directly: find the load voltage $\tilde{V}(0) = V_0^+(1+\Gamma)$ and compute $\frac{1}{2}|\tilde{V}(0)|^2/R_L$.

   P_L =

Documentation: _____