Learning Objectives 3.2 and 3.3: I can explain the physical meaning of and calculate the following quantities: propagation constant, phase constant, attenuation constant, characteristic impedance, VSWR, reflection coefficient, and transmission coefficient. I can use the transmission line equation to calculate the voltage/current at an arbitrary point on a specified transmission line.

Show your work. Write a **Documentation** line at the end (**None** if you did not collaborate). All lines are lossless.

1. **Coaxial line.** For a coaxial line with inner radius a, outer radius b, and a nonmagnetic dielectric of relative permittivity ϵ_r, $Z_0 \approx \frac{60}{\sqrt{\epsilon_r}} \ln \frac{b}{a}\ \Omega$.

   (a) A cable has $a = 0.29$ mm, $b = 1.85$ mm, and a polyethylene dielectric, $\epsilon_r = 2.25$. Find Z_0.

   Z_0 = $74.1\ \Omega$

   $$\ln \frac{b}{a} = \ln \frac{1.85}{0.29} = \ln 6.38 = 1.853$$
   $$Z_0 = \frac{60}{1.5} \times 1.853 = 74.1\ \Omega$$

   This is a $75\ \Omega$ cable, the kind used for video and cable television.

   (b) Find the phase velocity in this cable.

   v_p = 2×10^8 m/s

   $$v_p = \frac{c}{\sqrt{\epsilon_r}} = \frac{3 \times 10^8}{1.5} = 2 \times 10^8 \text{ m/s}$$

   (c) Design an air-filled coaxial line, $\epsilon_r = 1$, with $Z_0 = 50\ \Omega$ and an inner conductor of radius $a = 1$ mm. Find the radius b of the outer conductor.

   b = 2.30 mm

   $$\ln \frac{b}{a} = \frac{50}{60} = 0.833$$
   $$b = a\, e^{0.833} = 1 \times 2.30 = 2.30 \text{ mm}$$

   (d) We fill the line in part (c) with polyethylene, $\epsilon_r = 2.25$, without changing a or b. Find the new Z_0, and explain in one sentence why it went down.

   Z_0 = $33.3\ \Omega$

   $$Z_0 = \frac{60}{1.5} \times 0.833 = 33.3\ \Omega$$

   The dielectric raises the capacitance per meter by ϵ_r and leaves the inductance unchanged, so $Z_0 = \sqrt{L'/C'}$ falls by $\sqrt{\epsilon_r} = 1.5$.

2. **Reflection coefficient.** Each load below terminates a $50\ \Omega$ line.

(a) $Z_L = 25\ \Omega$. Find Γ and the VSWR.

$$\Gamma = \frac{25-50}{25+50} = -\frac{1}{3} = -0.333$$

$$\text{VSWR} = \frac{1+0.333}{1-0.333} = 2.0$$

The load is real and smaller than Z_0, so Γ is real and negative.

Γ = -0.333

VSWR = 2.0

(b) $Z_L = 30 - j40\ \Omega$. Find Γ in polar form and the VSWR.

$$\Gamma = \frac{-20-j40}{80-j40} = \frac{(-20-j40)(80+j40)}{80^2+40^2}$$

$$= \frac{-j4000}{8000} = -j0.5 = 0.5\angle -90^\circ$$

$$\text{VSWR} = \frac{1.5}{0.5} = 3.0$$

Γ = $0.5\angle -90^\circ$

VSWR = 3.0

(c) What fraction of the incident power does the load in part (b) reflect, and what fraction does it absorb?

$$|\Gamma|^2 = 0.5^2 = 0.25$$

$$1 - |\Gamma|^2 = 0.75$$

reflected = 25%

absorbed = 75%

(d) A load consists of a capacitor alone. Without computing, state $|\Gamma|$ and the VSWR, and explain why.

$|\Gamma| = 1$ and the VSWR is infinite. A capacitor stores energy but cannot dissipate any, so it returns all of the incident power; only the phase of Γ depends on its reactance.

3. **Reading a standing wave.** A probe slid along a $50\ \Omega$ RG-58 line at 300 MHz measures a largest voltage of 12 V and a VSWR of 3.

   (a) Find $|\Gamma|$.

$$|\Gamma| = \frac{\text{VSWR} - 1}{\text{VSWR} + 1} = \frac{2}{4} = 0.5$$

$|\Gamma|$ = **0.5**

   (b) Find the smallest voltage on the line and the amplitude of the incident wave, $|V_0^+|$.

$$V_{\text{min}} = \frac{V_{\text{max}}}{\text{VSWR}} = \frac{12}{3} = 4\ \text{V}$$

$$|V_0^+| = \frac{V_{\text{max}}}{1 + |\Gamma|} = \frac{12}{1.5} = 8\ \text{V}$$

V_{min} = **4 V**

$|V_0^+|$ = **8 V**

   (c) Find the distance between adjacent voltage minima. Signals travel along RG-58 at 2×10^8 m/s.

$$\lambda = \frac{2 \times 10^8}{3 \times 10^8} = 0.667\ \text{m}$$

$$\frac{\lambda}{2} = 33.3\ \text{cm}$$

spacing = **33.3 cm**

   (d) One voltage minimum falls exactly at the load, and the load is a pure resistance. Is Z_L larger or smaller than Z_0? Find Z_L.

   A minimum at the load means the reflected wave subtracts there, so $\theta_\Gamma = 180°$ and $\Gamma = -0.5$. The load is smaller than Z_0:

$$Z_L = \frac{Z_0}{\text{VSWR}} = \frac{50}{3} = 16.7\ \Omega$$

Z_L = **16.7 Ω**

4. **A complex load.** A load $Z_L = 100 + j100\,\Omega$ terminates a $50\,\Omega$ line.

(a) Find Γ in rectangular and polar form.

$$\Gamma = \frac{50 + j100}{150 + j100} = \frac{(50 + j100)(150 - j100)}{150^2 + 100^2}$$
$$= \frac{17{,}500 + j10{,}000}{32{,}500} = 0.538 + j0.308$$
$$= 0.620\angle 29.7^\circ$$

Γ = $0.620\angle 29.7^\circ$

(b) Find the VSWR.

$$\text{VSWR} = \frac{1.620}{0.380} = 4.27$$

VSWR = 4.27

(c) Find the distance from the load to the first voltage maximum, in wavelengths.

$$d_{\max} = \frac{\theta_\Gamma}{4\pi}\lambda = \frac{29.7^\circ}{720^\circ}\lambda = 0.041\lambda$$

$d_{\max}$ = 0.041λ

(d) Find the transmission coefficient τ in polar form.

$$\tau = 1 + \Gamma = 1.538 + j0.308 = 1.57\angle 11.3^\circ$$

τ = $1.57\angle 11.3^\circ$

5. **Is it worth matching?** A handheld radio feeds a $75\ \Omega$ antenna through $50\ \Omega$ coaxial cable.

   (a) Find Γ, the VSWR, and the fraction of the incident power the antenna absorbs.

$$\Gamma = \frac{75 - 50}{75 + 50} = 0.2$$

$$\text{VSWR} = \frac{1.2}{0.8} = 1.5$$

$$1 - |\Gamma|^2 = 1 - 0.04 = 0.96$$

Γ =	0.2
VSWR =	1.5
absorbed =	96%

   (b) A matching network would raise the absorbed power to 100%, but it adds parts, cost, and a little loss of its own. Using the VSWR rule of thumb from the lesson, would you add one? Justify your answer in one or two sentences.

   No. A VSWR of 1.5 is well below the rule-of-thumb limit of 2, and the mismatch costs only 4% of the power, about 0.18 dB. A matching network is likely to lose about as much as it recovers.

Documentation: _____