Learning Objective 3.2: I can explain the physical meaning of and calculate the following quantities: propagation constant, phase constant, attenuation constant, characteristic impedance, VSWR, reflection coefficient, and transmission coefficient.

Show your work. Write a **Documentation** line at the end (**None** if you did not collaborate).

1. **Characteristic impedance.** A lossless line has $L' = 300$ nH/m and $C' = 120$ pF/m.

   (a) Starting from $-d\tilde{V}/dz = (R' + j\omega L')\tilde{I}$ and $\tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$, derive $\mathsf{Z}_0 = \sqrt{\dfrac{R' + j\omega L'}{G' + j\omega C'}}$.

$$-\frac{d\tilde{V}}{dz} = \gamma V_0^+ e^{-\gamma z} - \gamma V_0^- e^{+\gamma z} = \left(R' + j\omega L'\right)\tilde{I}$$

$$\tilde{I}(z) = \frac{\gamma}{R' + j\omega L'}\left(V_0^+ e^{-\gamma z} - V_0^- e^{+\gamma z}\right)$$

$$\mathsf{Z}_0 = \frac{R' + j\omega L'}{\gamma} = \frac{R' + j\omega L'}{\sqrt{\left(R' + j\omega L'\right)\left(G' + j\omega C'\right)}} = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$$

   (b) Find Z_0 and v_p for this line.

$$\mathsf{Z}_0 = \sqrt{\frac{300 \times 10^{-9}}{120 \times 10^{-12}}} = 50\ \Omega$$

$$v_p = \frac{1}{\sqrt{300 \times 10^{-9} \times 120 \times 10^{-12}}} = 1.67 \times 10^8\ \text{m/s}$$

$\mathsf{Z}_0 =$ $50\ \Omega$

$v_p =$ 1.67×10^8 m/s

   (c) The voltage on the line has $V_0^+ = 10$ V and $V_0^- = 2$ V. Find I_0^+ and I_0^-.

$$I_0^+ = \frac{V_0^+}{\mathsf{Z}_0} = \frac{10}{50} = 0.2\ \text{A}$$

$$I_0^- = -\frac{V_0^-}{\mathsf{Z}_0} = -\frac{2}{50} = -0.04\ \text{A}$$

$I_0^+ =$ 0.2 A

$I_0^- =$ -0.04 A

   (d) A classmate measures a 1 m length of this line with an ohmmeter and expects to read $50\ \Omega$. Explain what Z_0 is, and why the ohmmeter does not read it.

Z_0 is the ratio of voltage to current in a single traveling wave on the line. It is not a resistance between the conductors: at DC the open line draws no current and the ohmmeter reads an open circuit. The line presents Z_0 at its input only to a signal that travels away and never returns, as on an infinitely long or matched line.

2. **Phase velocity and wavelength.** A 120 MHz signal on a lossless coaxial line has a phase constant $\beta = 4$ rad/m.

   (a) Starting from the phase of the incident wave, derive $v_p = \omega/\beta$.

   A point of constant phase satisfies $\omega t - \beta z =$ constant. Differentiating with respect to t:

   $$\omega - \beta \frac{dz}{dt} = 0 \quad \Longrightarrow \quad v_p = \frac{dz}{dt} = \frac{\omega}{\beta}$$

   (b) Find v_p and the wavelength.

   $$v_p = \frac{2\pi \times 1.2 \times 10^8}{4} = 1.88 \times 10^8 \text{ m/s}$$
   $$\lambda = \frac{2\pi}{4} = 1.57 \text{ m}$$

   v_p = 1.88×10^8 m/s

   λ = 1.57 m

   (c) For a coaxial line, $v_p = c/\sqrt{\epsilon_r}$. Find the relative permittivity of the dielectric.

   $$\epsilon_r = \left(\frac{c}{v_p}\right)^2 = \left(\frac{3 \times 10^8}{1.885 \times 10^8}\right)^2 = 2.53$$

   ϵ_r = 2.53

3. **Low-loss attenuation.** A $50\,\Omega$ cable at 200 MHz has $R' = 2\,\Omega/\text{m}$, $L' = 250$ nH/m, $G' = 1 \times 10^{-5}$ S/m, and $C' = 100$ pF/m.

   (a) Check that the low-loss approximation applies.

$$\frac{R'}{\omega L'} = \frac{2}{2\pi \times 2 \times 10^8 \times 250 \times 10^{-9}} = \frac{2}{314} = 0.006$$
$$\frac{G'}{\omega C'} = \frac{1 \times 10^{-5}}{0.126} = 8 \times 10^{-5}$$

   Both are far below one.

   (b) Find α in Np/m and in dB/m.

$$\alpha = \frac{R'}{2Z_0} + \frac{G'Z_0}{2} = \frac{2}{100} + \frac{1 \times 10^{-5} \times 50}{2}$$
$$= 0.02 + 0.00025 = 0.0203 \text{ Np/m}$$
$$= 8.686 \times 0.0203 = 0.176 \text{ dB/m}$$

α = 0.0203 Np/m

α = 0.176 dB/m

   (c) A 25 m run of this cable feeds an antenna. Find the loss in dB and the fraction of the input power that reaches the antenna.

$$\text{loss} = 0.176 \times 25 = 4.4 \text{ dB}$$
$$\frac{P_{\text{out}}}{P_{\text{in}}} = 10^{-4.4/10} = 0.36$$

loss = 4.4 dB

fraction = 36%

   (d) We use the same cable at 800 MHz. Conductor loss scales as the square root of frequency, and dielectric loss in proportion to it. Estimate the new α in dB/m, and state which mechanism dominates.

   Frequency rises by a factor of 4, so the conductor term doubles and the dielectric term quadruples:

$$\alpha = 2 \times 0.02 + 4 \times 0.00025 = 0.041 \text{ Np/m} = 0.36 \text{ dB/m}$$

   The conductor loss still dominates.

Documentation: _____