Learning Objective 3.1: I can describe the origin of the transmission line equation, understand the method of developing a solution, and interpret the solution.

Show your work. Write a **Documentation** line at the end (**None** if you did not collaborate).

1. **From the telegrapher's equations to the wave equation.** Start from the telegrapher's equations,

$$-\frac{\partial v(z,t)}{\partial z} = R'\, i(z,t) + L'\frac{\partial i(z,t)}{\partial t}$$
$$-\frac{\partial i(z,t)}{\partial z} = G'\, v(z,t) + C'\frac{\partial v(z,t)}{\partial t}$$

(a) With $v(z,t) = \text{Re}\left[\widetilde{V}(z)e^{j\omega t}\right]$, show that the phasor of $\partial v/\partial t$ is $j\omega\widetilde{V}(z)$.

(b) Write both telegrapher's equations in phasor form.

(c) Uncouple the equations to derive the wave equation for the current, $\widetilde{I}(z)$. Show each step.

(d) Substitute the trial solution $\widetilde{I}(z) = Ae^{sz}$ into your result, find the characteristic equation and its roots, and write the general solution.

2. **The propagation constant of a lossy line.** A line has $R' = 0.5\,\Omega/\text{m}$, $L' = 250$ nH/m, $G' = 0$, and $C' = 100$ pF/m, and carries a 10 MHz signal.

   (a) Find the series impedance per meter, $R' + j\omega L'$, and the shunt admittance per meter, $G' + j\omega C'$.

   Z' =

   Y' =

   (b) Find $\gamma = \alpha + j\beta$.

   α =

   β =

   (c) Find the wavelength on the line.

   λ =

   (d) Compare β with its value on the same line with $R' = 0$. What does the comparison say about the effect of a small loss on the phase constant?

3. **Reading a solution.** The voltage on a line is

$$v(z,t) = 5\,e^{-0.02z}\cos\left(2\pi \times 10^8\,t - 3z + 30^\circ\right) + 2\,e^{0.02z}\cos\left(2\pi \times 10^8\,t + 3z\right)\ \text{V}$$

with z in meters.

(a) Which term travels toward $+z$ and which toward $-z$? Explain how you can tell.

(b) Find α, β, and the wavelength.

α = ____

λ = ____

(c) Write the phasor constants V_0^+ and V_0^-.

V_0^+ = ____

V_0^- = ____

(d) The second term grows as $e^{0.02z}$. Explain why this does not mean the reflected wave gains energy.

4. **The lossless line.** RG-58 has $L' = 259$ nH/m and $C' = 96.8$ pF/m.

   (a) Set $R' = G' = 0$ and show that γ is purely imaginary.

   (b) Find β and the wavelength at 50 MHz.

   $\beta =$ ____

   $\lambda =$ ____

   (c) In the time domain, describe what the incident wave on a lossless line looks like to an observer at a fixed position z, and to an observer moving along with one of its crests.

Documentation: _____