Learning Objective 3.1: I can describe the origin of the transmission line equation, understand the method of developing a solution, and interpret the solution.

Show your work. Write a **Documentation** line at the end (**None** if you did not collaborate). Unless a problem says otherwise, signals travel along RG-58 coaxial cable at $v_p = 2 \times 10^8$ m/s.

1. **Electrical length.** A 3 m length of RG-58 connects a signal generator to a load.

   (a) Find the wavelength in the cable at 50 kHz and the electrical length ℓ/λ. Does the cable behave as an ideal wire?

   $\lambda =$ ____

   $\ell/\lambda =$ ____

   (b) Repeat part (a) at 150 MHz.

   $\lambda =$ ____

   $\ell/\lambda =$ ____

   (c) At 150 MHz, find the phase delay from one end of the cable to the other, in degrees, and the time delay.

   $\Delta\phi =$ ____

   $\tau =$ ____

   (d) Using the $\lambda/10$ rule of thumb, find the highest frequency at which this cable still behaves as an ideal wire.

   $f_{\text{max}} =$ ____

2. **Wire or transmission line?**

(a) A microstrip trace on a circuit board is 5 cm long and carries a 1 GHz clock. Signals on the board travel at $v_p = 1.65 \times 10^8$ m/s. Find ℓ/λ and decide which model applies.

$\ell/\lambda =$ ____

(b) A 10 m speaker cable carries audio up to 20 kHz at $v_p = 2 \times 10^8$ m/s. Find ℓ/λ at the highest audio frequency and decide which model applies.

$\ell/\lambda =$ ____

(c) The speaker cable in part (b) is 200 times longer than the circuit-board trace in part (a). Explain in one or two sentences why the short trace needs the transmission line model and the long cable does not.

3. **The four elements.** The lumped-element model places $R'\Delta z$ and $L'\Delta z$ in series and $G'\Delta z$ and $C'\Delta z$ in shunt in each section of the line.

   (a) For each of R', L', G', and C', give its units and the physical effect in the line that it represents.

   (b) Which elements are zero on a lossless line, and why?

   (c) RG-58 has $L' = 259$ nH/m and $C' = 96.8$ pF/m. Find the total inductance and capacitance of a 1.5 m length.

   L = ____

   C = ____

   (d) A classmate claims that $G' = 1/R'$, since conductance is the inverse of resistance. Explain why the claim is wrong.

4. **Kirchhoff's laws on one section.** Use the section in the lesson, with $v(z,t)$ and $i(z,t)$ at the input and $v(z+\Delta z,t)$ and $i(z+\Delta z,t)$ at the output, and the shunt elements connected at the output side of the series elements.

   (a) Write Kirchhoff's voltage law around the loop from the input terminals to the output terminals.

   (b) Write Kirchhoff's current law at the node where the shunt elements connect.

   (c) We choose $\Delta z = \lambda/20$ for a model of RG-58 at 500 MHz. Find Δz and the number of sections needed to model a 1 m cable.

   Δz = [ ]

   N = [ ]

   (d) Starting from your result in part (a), divide by Δz and take the limit as $\Delta z \to 0$ to derive the first telegrapher's equation. Name the step where the derivative in z appears.

5. **The telegrapher's equations.** The two telegrapher's equations for a line with all four elements are

$$-\frac{\partial v(z,t)}{\partial z} = R'\, i(z,t) + L'\frac{\partial i(z,t)}{\partial t}$$
$$-\frac{\partial i(z,t)}{\partial z} = G'\, v(z,t) + C'\frac{\partial v(z,t)}{\partial t}$$

   (a) In the current law for one section, the shunt elements see $v(z+\Delta z, t)$, not $v(z,t)$. Explain why the second equation contains $v(z,t)$.

   (b) Write both equations for a lossless line.

   (c) The equations are called *coupled*. Explain what that means, and why we cannot solve the first equation for $v(z,t)$ by itself.

   (d) On a lossless line carrying a DC voltage and current that do not change in time, what do the equations say about v and i along the line? Explain the result physically.

Documentation: _____