Learning Objectives 2.1 and 2.2: I can recall the definition of scalar fields, vector fields, phasors, and Euler's identity. I can explain the basic terms and quantities used to describe electromagnetic fields.

Show your work. Write a **Documentation** line at the end (**None** if you did not collaborate).

1. **Euler's identity.**

   (a) Starting from $e^{j\theta} = \cos\theta + j\sin\theta$, derive $\cos\theta = \dfrac{e^{j\theta} + e^{-j\theta}}{2}$.

   Replace θ with $-\theta$, using $\cos(-\theta) = \cos\theta$ and $\sin(-\theta) = -\sin\theta$, then add the two:

   $$e^{-j\theta} = \cos\theta - j\sin\theta$$
   $$e^{j\theta} + e^{-j\theta} = 2\cos\theta$$

   (b) Write $e^{j\pi/3}$ in rectangular form, and find its magnitude.

   $$e^{j\pi/3} = \cos 60^\circ + j\sin 60^\circ = 0.5 + j0.866$$
   $$\left|e^{j\pi/3}\right| = \sqrt{0.5^2 + 0.866^2} = 1$$

   $e^{j\pi/3}$ = $0.5 + j0.866$

   magnitude = 1

   (c) Write $2e^{-j\pi/4}$ in rectangular form.

   $$2e^{-j\pi/4} = 2\cos 45^\circ - j2\sin 45^\circ = 1.414 - j1.414$$

   $2e^{-j\pi/4}$ = $1.414 - j1.414$

2. **Phasors.** Use the cosine reference, $v(t) = \text{Re}\left[\tilde{V}e^{j\omega t}\right]$.

(a) Find the phasor of $v(t) = 10\cos(2\pi \times 60\,t - 45^\circ)$ V.

$$\tilde{V} = 10e^{-j45^\circ} = 10\angle{-45^\circ}\ \text{V}$$

$\tilde{V}$ = $10\angle{-45^\circ}$ V

(b) Find the phasor of $i(t) = 2\sin(\omega t + 30^\circ)$ A.

Write the sine as a cosine first:

$$i(t) = 2\cos(\omega t + 30^\circ - 90^\circ) = 2\cos(\omega t - 60^\circ)$$
$$\tilde{I} = 2\angle{-60^\circ}\ \text{A}$$

$\tilde{I}$ = $2\angle{-60^\circ}$ A

(c) Use phasors to write $3\cos\omega t + 4\sin\omega t$ as a single cosine.

$$\tilde{V} = 3\angle{0^\circ} + 4\angle{-90^\circ} = 3 - j4 = 5\angle{-53.1^\circ}$$
$$v(t) = 5\cos(\omega t - 53.1^\circ)$$

$v(t)$ = $5\cos(\omega t - 53.1^\circ)$

(d) Find the phasor of dv/dt for the voltage in part (a).

Differentiating in time multiplies the phasor by $j\omega$, and $j = 1\angle{90^\circ}$:

$$j\omega\tilde{V} = (2\pi \times 60)\angle{90^\circ} \times 10\angle{-45^\circ}$$
$$= 3770\angle{45^\circ}\ \text{V/s}$$

phasor = $3770\angle{45^\circ}$ V/s

3. **A sinusoidal wave.** A wave on a line is $y(x,t) = 0.5\cos\left(2\pi \times 10^8\, t - 2.094\, x\right)$, with x in meters.

   (a) Find the frequency, wavelength, and phase velocity.

$$f = \frac{\omega}{2\pi} = 10^8 \text{ Hz} = 100 \text{ MHz}$$
$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{2.094} = 3.00 \text{ m}$$
$$v_p = f\lambda = 10^8 \times 3.00 = 3.0 \times 10^8 \text{ m/s}$$

f =	100 MHz
λ =	3.00 m
v_p =	3.0×10^8 m/s

   (b) In which direction does the wave travel? Show it by writing the argument of the cosine as a function of $x - v_p t$ or $x + v_p t$.

$$\omega t - \beta x = -\beta\left(x - \frac{\omega}{\beta} t\right) = -\beta\left(x - v_p t\right)$$

The argument is a function of $x - v_p t$, so the wave travels in the $+x$ direction.

4. **Field quantities and materials.** Use $\epsilon_0 = 8.854 \times 10^{-12}$ F/m, $\mu_0 = 4\pi \times 10^{-7}$ H/m, and $e = 1.602 \times 10^{-19}$ C.

   (a) An electric field of 1 kV/m exists in polyethylene, $\epsilon_r = 2.25$. Find the electric flux density, and the magnitude of the force on an electron in the field.

   $D =$ 1.99×10^{-8} C/m^2

   $F =$ 1.60×10^{-16} N

   $$D = \epsilon_r \epsilon_0 E = 2.25 \times 8.854 \times 10^{-12} \times 10^3$$
   $$= 1.99 \times 10^{-8} \text{ C/m}^2$$
   $$F = eE = 1.602 \times 10^{-19} \times 10^3 = 1.60 \times 10^{-16} \text{ N}$$

   (b) A field of 0.01 V/m drives current in copper, $\sigma = 5.8 \times 10^7$ S/m. Find the current density.

   $J =$ 5.8×10^5 A/m^2

   $$J = \sigma E = 5.8 \times 10^7 \times 0.01 = 5.8 \times 10^5 \text{ A/m}^2$$

   (c) A magnetic field intensity of 100 A/m exists in air. Find the magnetic flux density.

   $B =$ 1.26×10^{-4} T

   $$B = \mu_0 H = 4\pi \times 10^{-7} \times 100 = 1.26 \times 10^{-4} \text{ T}$$

   (d) Show that $c = 1/\sqrt{\mu_0 \epsilon_0}$ gives the speed of light, and explain in one sentence what it says about the relationship between the electric and magnetic properties of free space and light.

   $$c = \frac{1}{\sqrt{4\pi \times 10^{-7} \times 8.854 \times 10^{-12}}} = 3.00 \times 10^8 \text{ m/s}$$

   The speed of light is set entirely by the two constants that describe how free space responds to electric and magnetic fields, because light is an electromagnetic wave.

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