ECE343 Spring 2025

Objective 9.5:
UPW Boundaries



UNITED STATES
AIR FORCE
ACADEMY

SCHEDULE

Lesson	Topics	Reading	Notes
24	BVP (Capacitor, Coax), Electric Potential	5.16	LO 5.3
25	Electric Boundary Conditions	5.18-5.22	LO 5.4
26	BC's/Capacitance	5.23-5.24	
27	Electrostatic Lab	Lab packet	
Chapter 6: Steady Current & Conductivity			
28	Current density, Conductivity, Resistance	Ch. 6	
Chapter 7: Magnetostatics			
29	Magnetic Fields, Gauss' Law for Magnetism	7.1-7.3	LO 7.1-7.2 **GR 2**
30	Ampere's Law	7.4-7.5, 7.9	LO 7.3
31	Ampere's Law Applied	7.6-7.8	LO 7.3
32	Magnetic Boundary Conditions	7.10-7.14	LO 7.4
33	Magnetostatics Lab	Lab packet	
Chapter 8: Time-Varying Fields			
34	Faraday's Law, Displacement Current, Maxwell Eqns	8.3, 8.8, 8.9	LO 8.1
35	Faraday's Law, Displacement Current, Maxwell Eqns	8.3, 8.8, 8.9	LO 8.2-8.3
Chapter 9: Plane Waves in Lossless Media			
36	Maxwell's Equations in lossless, source-free region	9.1-9.2	LO 9.1
37	Maxwell's Equations in lossless, source-free region	9.1-9.2	LO 9.1
38	Uniform Plane Waves & Polarization	9.4-9.6	LO 9.2-9.3
39	Normal Incidence	Vol 2, 5.6-5.7	LO 9.4-9.5
40	Oblique Incidence	Vol 2, 5.6-5.7	LO 9.5
41	Oblique Incidence	Vol 2, 5.6-5.7	LO 9.5
42	Incidence Lab	TBD	LO 9.5

Ch 9 Learning Objectives

9.1 I understand the process of decoupling and finding solutions for Maxwell's equations for dynamic fields.

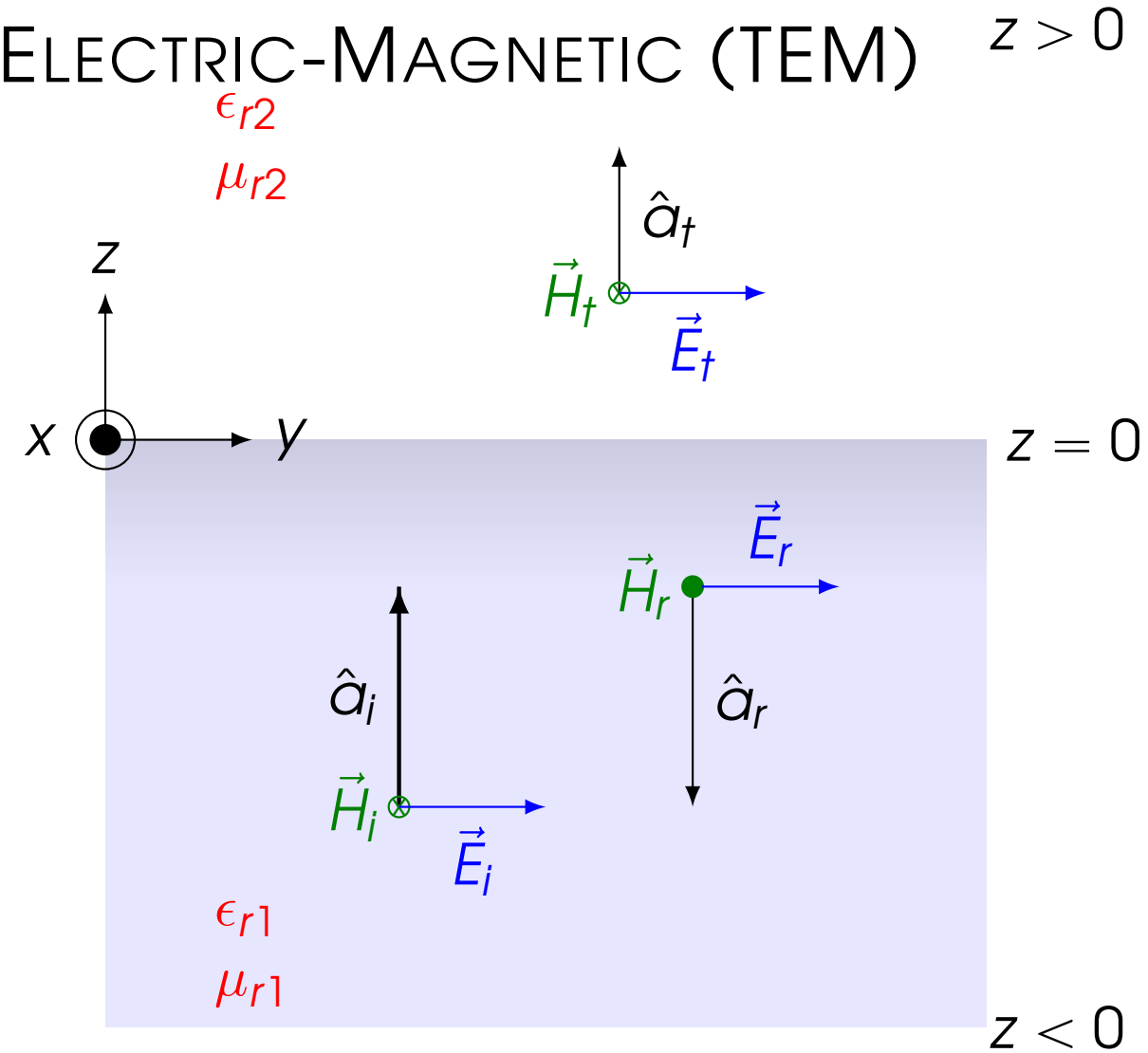
9.2 I can articulate what a Uniform Plane Wave (UPW) is and describe key properties that relate the electric and magnetic field.

9.3 I can calculate the electric and magnetic fields for a UPW.

9.4 I can articulate the concept of polarization and determine the polarization for various UPWs.

9.5 I can determine the reflection and transmission of UPW at a boundary for Transverse Electric (TE) and Transverse Magnetic (TM) waves, using boundary conditions.

Normal Incidence: Transverse Electric-Magnetic (TEM)



Incident Fields:

$$\tilde{E}^i = \hat{\mathbf{y}} E^i_{y0} e^{-\alpha_1 z} e^{-j\beta_1 z}$$

$$\tilde{H}^i = -\hat{\mathbf{x}} \frac{E^i_{y0}}{\eta_1} e^{-\alpha_1 z} e^{-j\beta_1 z}$$

Reflected Fields:

$$\tilde{E}^r = \hat{\mathbf{y}} E^r_{y0} e^{\alpha_1 z} e^{j\beta_1 z}$$

$$\tilde{H}^r = \hat{\mathbf{x}} \frac{E^r_{y0}}{\eta_1} e^{\alpha_1 z} e^{j\beta_1 z}$$

Transmitted Fields:

$$\tilde{E}^t = \hat{\mathbf{y}} E^t_{y0} e^{-\alpha_2 z} e^{-j\beta_2 z}$$

$$\tilde{H}^t = -\hat{\mathbf{x}} \frac{E^t_{y0}}{\eta_2} e^{-\alpha_2 z} e^{-j\beta_2 z}$$

Normal Incidence (TEM): BC's

Use boundary conditions, assuming we know incident E-field

$$\vec{E}_{t1} = \vec{E}_{t2}$$

$$E^i_{y0} + E^r_{y0} = E^t_{y0}$$

$$\hat{n}_{21} \times \left(\vec{H}_1 - \vec{H}_2\right) = \cancel{\vec{J}_s}^{\,0}$$

$$-\hat{\mathbf{z}} \times \left(\hat{\mathbf{y}}\frac{E^i_{y0}}{\eta_1} + \hat{\mathbf{y}}\frac{E^r_{y0}}{\eta_1} - \hat{\mathbf{y}}\frac{E^t_{y0}}{\eta_2}\right) = 0$$

2 eqns, 2 unkns (E^t_{x0} & E^r_{x0}):

$$\Gamma = \frac{E^r_{x0}}{E^i_{x0}} = \ldots = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$T = \ldots = \frac{E^t_{x0}}{E^i_{x0}} = 1 + \Gamma = \frac{2\eta_2}{\eta_2 + \eta_1}$$

Now, for dielectric ($\sigma = 0$):

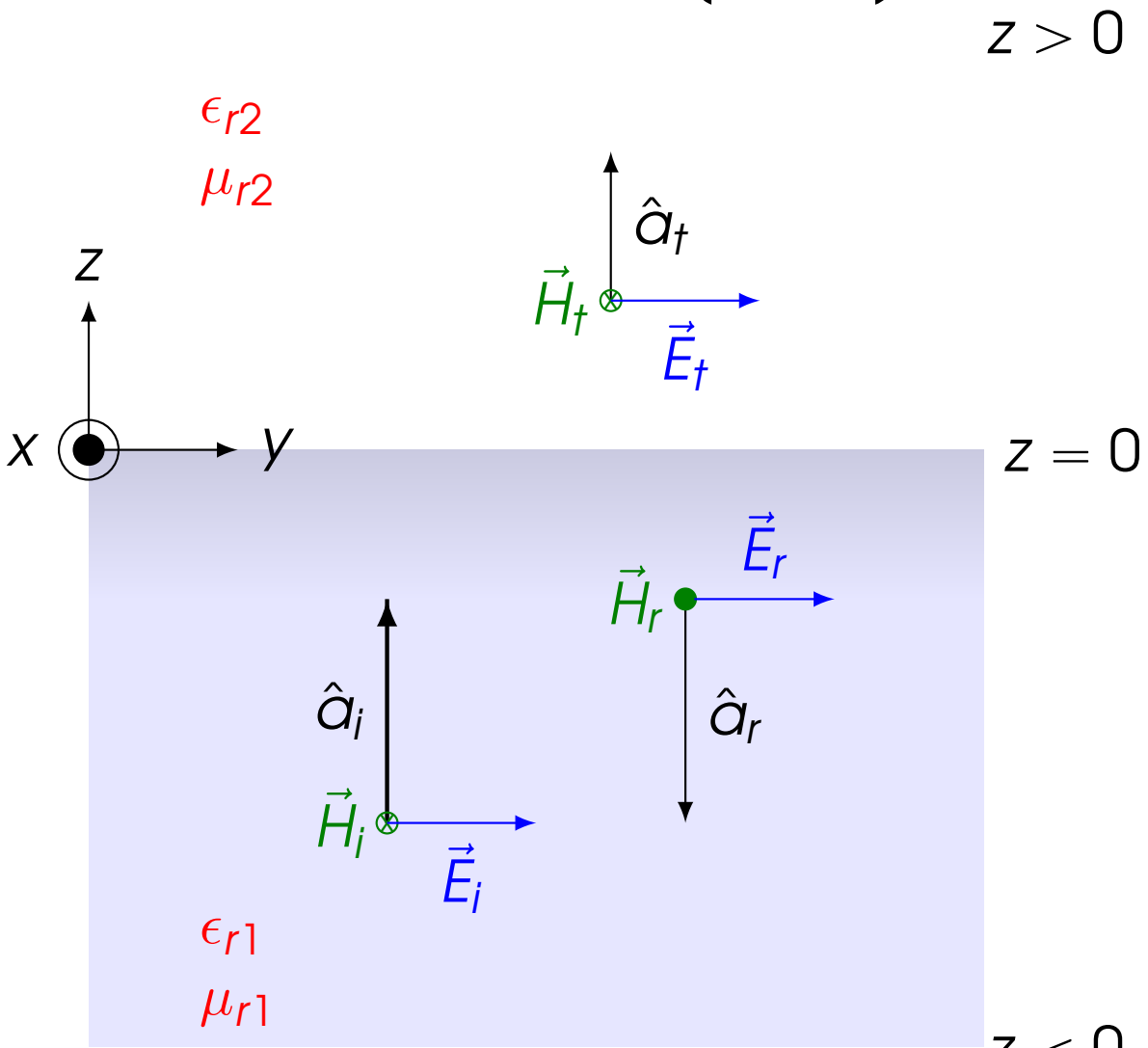
$$\eta_k = 120\pi\sqrt{\frac{\mu_{r,k}}{\varepsilon_{r,k}}}$$

$$\gamma = \sqrt{j\omega\mu\left(\sigma + j\omega\varepsilon\right)} = j\omega\sqrt{\mu\varepsilon}$$

$$= \alpha + j\beta$$

$$\alpha = 0 \qquad \beta = \omega\sqrt{\mu\varepsilon}$$

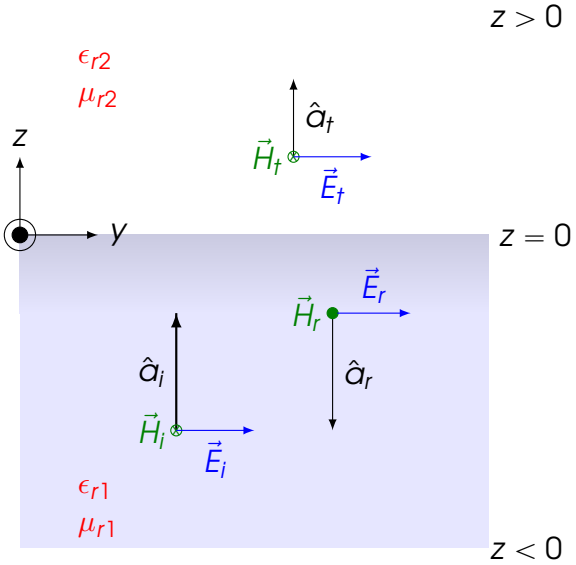
Normal Incidence (TEM): Example



$E^i_{y0} = 1$ V/m, $\varepsilon_{r1} = 1, \varepsilon_{r2} = 2$

$$\gamma = \sqrt{j\omega\mu\,(0 + j\omega\varepsilon)}$$
$$= j\omega\sqrt{\mu\varepsilon}$$
$$\Longrightarrow \alpha = 0,\ \beta = \omega\sqrt{\mu\varepsilon}$$
$$\beta_1 = \omega\left(3.34 \times 10^{-9}\right)$$
$$\beta_2 = \omega\left(4.72 \times 10^{-9}\right)$$
$$\eta_1 = \frac{120\pi}{\sqrt{1}} = 377\Omega$$
$$\eta_2 = \frac{120\pi}{\sqrt{2}} = 267\Omega$$
$$\Gamma_{\text{TEM}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$
$$= \frac{267 - 377}{267 + 377} = -0.171$$
$$T = 1 + \Gamma_{\text{TEM}} = 0.829$$

Normal Incidence (TEM): Example (cont'd)



Incident Fields:

$$\widetilde{E}^i = \hat{\mathbf{y}} E^i_{y0} e^{-\alpha_1 z} e^{-j\beta_1 z} \hat{\mathbf{y}}\,(1\text{V/m})\, e^0 e^{-j\beta_1 z}$$

$$\widetilde{H}^i = -\hat{\mathbf{x}} \frac{E^i_{y0}}{\eta_1} e^{-\alpha_1 z} e^{-j\beta_1 z} = -\hat{\mathbf{x}} \frac{1}{377} e^0 e^{-j\beta_1 z}$$

$$= -\hat{\mathbf{x}}\,(2.54\text{mA/m})\, e^{-j\beta_1 z}$$

Reflected Fields:

$$\widetilde{E}^r = \hat{\mathbf{y}} E^t_{y0} \Gamma_{\text{TEM}} e^{\alpha_1 z} e^{j\beta_1 z} = (1\text{V/m})\,(-0.171)\, e^{j\beta_1 z}$$

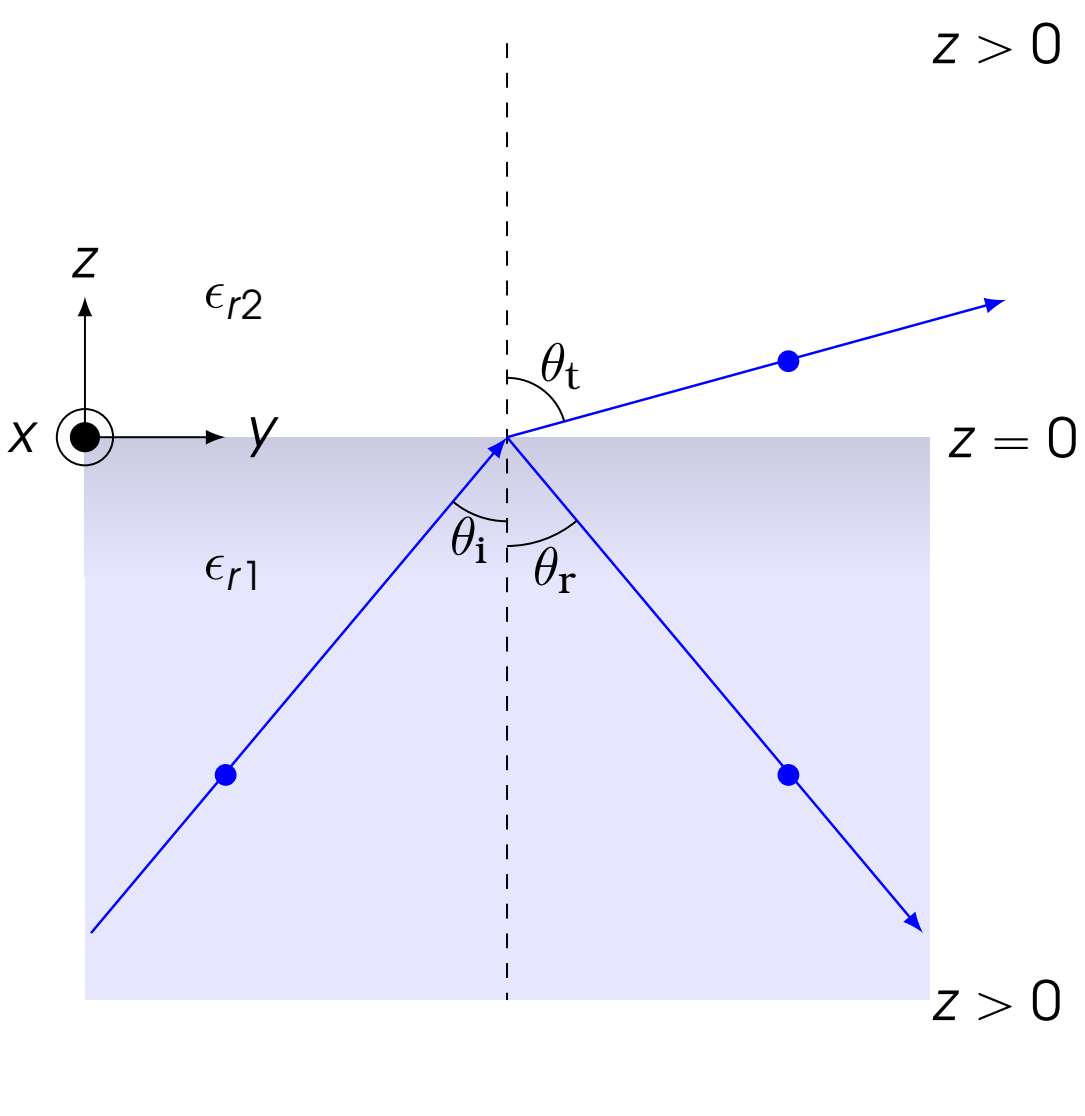
$$\widetilde{H}^r = \hat{\mathbf{x}} \frac{-0.171}{377} e^{j\beta_1 z} = \hat{\mathbf{x}}\,(0.454\text{mA/m})\, e^{j\beta_1 z}$$

Transmitted Fields:

$$\widetilde{E}^t = \hat{\mathbf{y}} E^i_{y0} T e^{-\alpha_2 z} e^{-j\beta_2 z} = \hat{\mathbf{y}}\,(0.829\text{V/m})\, e^{-j\beta_2 z}$$

$$\widetilde{H}^t = -\hat{\mathbf{x}} \frac{E^t_{y0}}{\eta_2} e^{-\alpha_2 z} e^{-j\beta_2 z} = -\hat{\mathbf{x}}\,(3.10\text{mA/m})\, e^{-j\beta_2 z}$$

Oblique Incidence: Transverse Electric (TE)



$$\tilde{E}^i = \hat{\mathbf{x}} E^i_{x0} e^{-j(\vec{\beta}_1 \cdot \vec{a}_i)}$$

$$\hat{a}_\rho = \hat{\mathbf{y}} y + \hat{\mathbf{z}} z$$

$$\vec{\beta} = \hat{\mathbf{y}} \beta_y + \hat{\mathbf{z}} \beta_z$$

$$= \hat{\mathbf{y}} \beta_1 \sin(\theta_i) + \hat{\mathbf{z}} \beta_1 \cos\left(\theta^i\right)$$

$$\tilde{E}^i = \hat{\mathbf{x}} E^i_{x0} e^{-j\beta_1 (y \sin(\theta^i) + z \cos(\theta^i))}$$

$$\tilde{H}^i = \frac{E^i_{x0}}{\eta_1} \left[\hat{\mathbf{y}} \cos\left(\theta^i\right) - \hat{\mathbf{z}} \sin\left(\theta^i\right)\right] e^{-j\beta_1 [x \sin(\theta^i) + z \cos(\theta^i)]}$$

Oblique Incidence (TE)

$$\widetilde{E}^r = \hat{\mathbf{y}} E^r_{y0} e^{-j\beta_1\left(x\sin\left(\theta^r\right)+z\cos\left(\theta^r\right)\right)}$$

$$\widetilde{H}^r = \frac{E^r_{y0}}{\eta_1}\left[-\hat{\mathbf{x}}\cos\left(\theta^r\right) - \hat{\mathbf{z}}\sin\left(\theta^i\right)\right] e^{-j\beta_1\left[y\sin\left(\theta^r\right)+z\cos\left(\theta^r\right)\right]}$$

$$\widetilde{E}^t = \hat{\mathbf{y}} E^t_{y0} e^{j\left(\vec{\beta}\cdot\vec{a}_\rho\right)}$$

$$\widetilde{H}^t = \frac{E^t_{y0}}{\eta_2}\left[-\hat{\mathbf{x}}\cos\left(\theta^t\right) + \hat{\mathbf{z}}\sin\left(\theta^t\right)\right] e^{-j\beta_1\left[y\sin\left(\theta^t\right)+z\cos\left(\theta^t\right)\right]}$$

Oblique Incidence (TE)

Use transverse (E-field) BCs to determine θ^r and θ^t:

$$\widetilde{E}_{t1}(z=0,t) = \widetilde{E}_{t2}(z=0,t)$$

$$\hat{\mathbf{y}}E^i_{y0}e^{-j\beta_1\left(x\sin\left(\theta^i\right)\right)} + \hat{\mathbf{y}}E^r_{y0}e^{-j\beta_1\left(x\sin\left(\theta^r\right)\right)} = \hat{\mathbf{y}}E^t_{y0}e^{-j\beta_1\left(x\sin\left(\theta^t\right)\right)}$$

"Phase Matching" $\Longrightarrow$

$$-j\beta_1 x\sin\left(\theta^i\right) = -j\beta_1 x\sin\left(\theta^r\right) = -j\beta_2 x\sin\left(\theta^t\right)$$

$$\theta^i = \theta^r$$

$$\beta_1\sin\left(\theta^i\right) = \beta_2\sin\left(\theta^p\right)$$

$$\theta^t = \sin^{-1}\left[\frac{\beta_1}{\beta_2}\sin\left(\theta^i\right)\right]$$

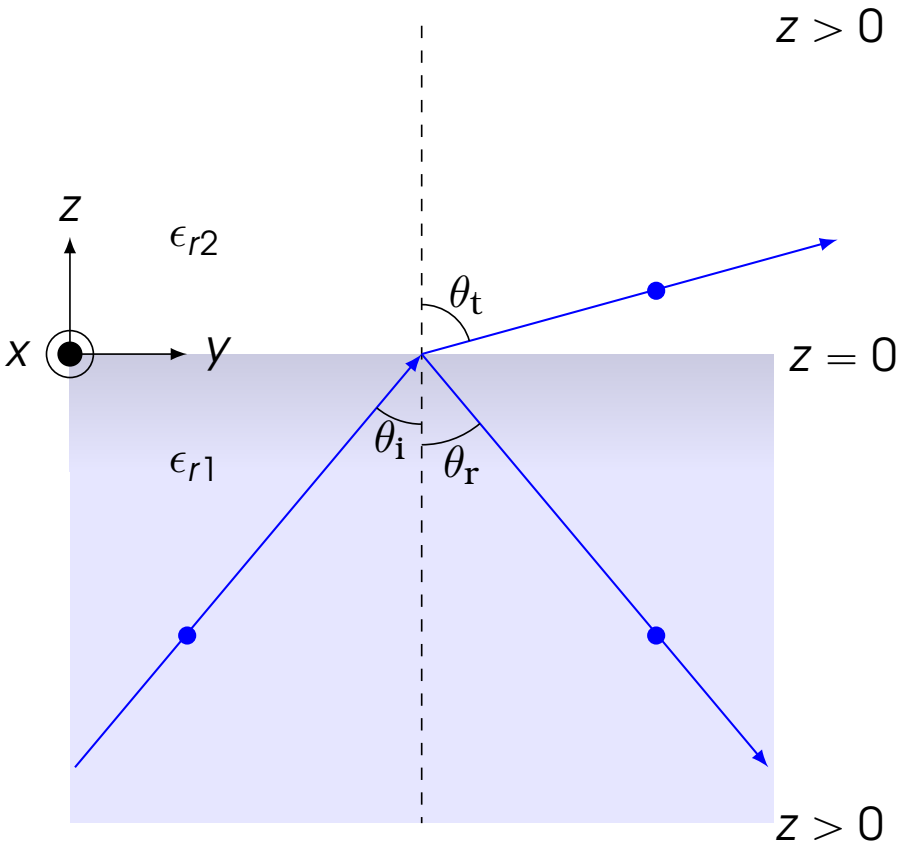
Oblique Incidence (TE)

Use transverse (H-field) BCs to determine Γ_{TE} and T_{TE}:

$$\hat{a}_{21} \times \left(\widetilde{H}_1 - \widetilde{H}_2\right) = \widetilde{J} \qquad \text{with: } \Gamma = \frac{E^r_{y0}}{E^i_{y0}}$$

$$\Gamma_{TE} = \frac{E^r_{y0}}{E^i_{y0}} = \frac{\eta_2 \cos\left(\theta^i\right) - \eta_1 \cos\left(\theta^t\right)}{\eta_2 \cos\left(\theta^i\right) + \eta_1 \cos\left(\theta^t\right)}$$

Oblique Incidence (TE): Summary



TE Summary

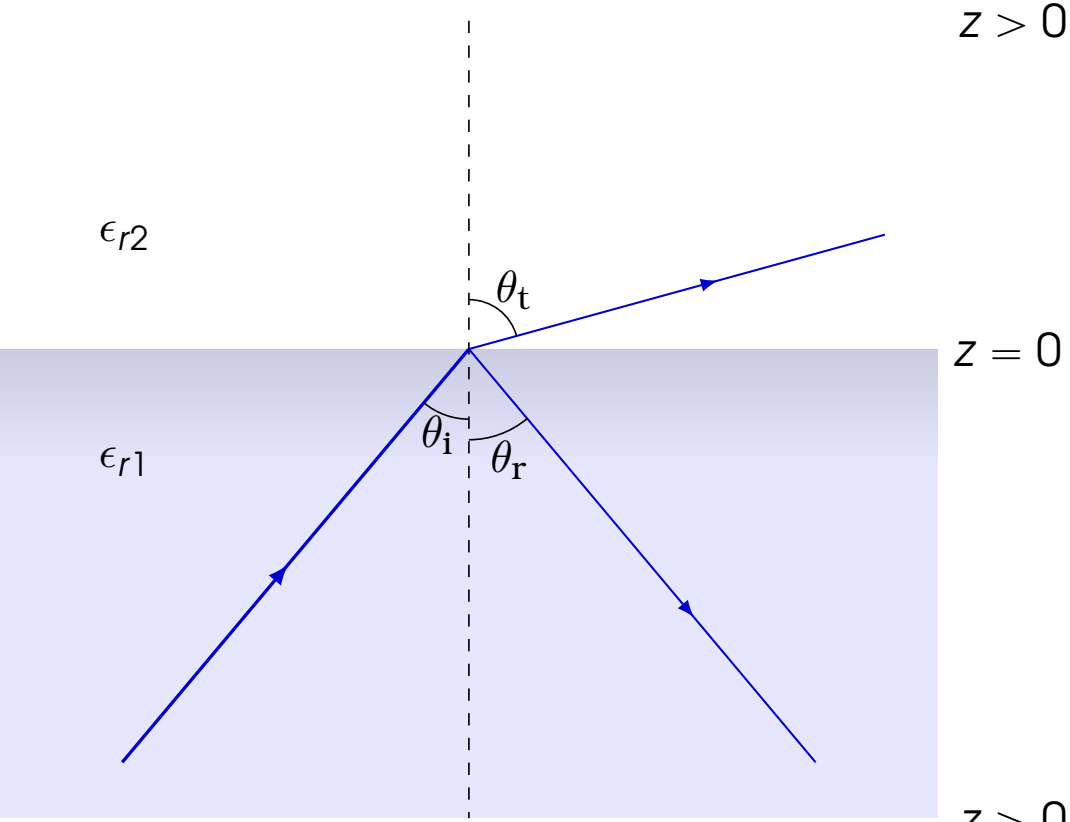
$$\theta^t = \sin^{-1}\left[\frac{\beta_1}{\beta_2}\sin\left(\theta^i\right)\right]$$

$$\theta^r = \theta^i$$

$$\Gamma_{TE} == \frac{\eta_2\cos\left(\theta^i\right) - \eta_1\cos\left(\theta^t\right)}{\eta_2\cos\left(\theta^i\right) + \eta_1\cos\left(\theta^t\right)}$$

$$T_{TE} = \frac{2\eta_2\cos\left(\theta^i\right)}{\eta_2\cos\left(\theta^i\right) + \eta_1\cos\left(\theta^t\right)}$$

Oblique Incidence (TM)



TM Summary

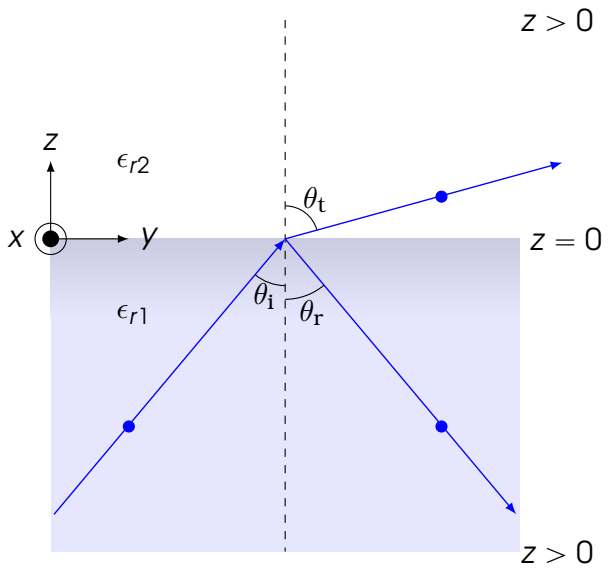
$$\theta^t = \sin^{-1}\left[\frac{\beta_1}{\beta_2}\sin\left(\theta^i\right)\right]$$

$$\theta^r = \theta^i$$

$$\Gamma_{TM} = \frac{\eta_2\cos\left(\theta^t\right) - \eta_1\cos\left(\theta^i\right)}{\eta_2\cos\left(\theta^t\right) + \eta_1\cos\left(\theta^i\right)}$$

$$T_{TM} = \frac{2\eta_2\cos\left(\theta^i\right)}{\eta_1\cos\left(\theta^t\right) + \eta_2\cos\left(\theta^i\right)}$$

Oblique Incidence (TM): Example



$$\theta^t = \sin^{-1}\left[\frac{\beta_1}{\beta_2}\sin\left(\theta^i\right)\right]$$

$$\theta^r = \theta^i$$

$$\Gamma_{TM} = \frac{E^r_{y0}}{E^i_{y0}} = \frac{\eta_2\cos\left(\theta^t\right) - \eta_1\cos\left(\theta^i\right)}{\eta_2\cos\left(\theta^t\right) + \eta_1\cos\left(\theta^i\right)}$$

$$T_{TM} = \frac{2\eta_2\cos\left(\theta^i\right)}{\eta_1\cos\left(\theta^i\right) + \eta_2\cos\left(\theta^t\right)}$$

$$\tilde{E}^i = \hat{\mathbf{y}}E_{oy}e^{-j\beta_1\left(x\sin\left(\theta^i\right)+z\cos\left(\theta_i\right)\right)}$$

$$\tilde{H} \Longrightarrow \frac{1}{\eta_1}\vec{a}_p \times \tilde{E} = \frac{1}{\eta_1}\left(\hat{\mathbf{x}} + \hat{\mathbf{z}}\right) \times \hat{\mathbf{y}}$$

$$= \frac{1}{\eta_1}\left(\hat{\mathbf{z}} - \hat{\mathbf{x}}\right)$$

$$\tilde{H} = \frac{E_{oy}}{\eta_1}\left(\hat{\mathbf{x}}\cos\left(\theta^i\right) + \hat{\mathbf{z}}\sin\left(\theta^i\right)\right)$$
$$e^{-j\beta_1\left(x\sin\left(\theta^i\right)+z\cos\left(\theta_i\right)\right)}$$

$$\tilde{E}^r = \hat{\mathbf{y}}\Gamma_{TE}E_{oy}$$
$$e^{-j\beta_1\left(x\sin\left(\theta^i\right)-z\cos\left(\theta_i\right)\right)}$$

$$\tilde{H}^r = \frac{\Gamma_{TE}E_{oy}}{\eta_1}\left(\hat{\mathbf{x}}\cos\left(\theta^i\right) + \hat{\mathbf{z}}\sin\left(\theta^i\right)\right)$$
$$e^{-j\beta_1\left(x\sin\left(\theta^i\right)-z\cos\left(\theta_i\right)\right)}$$

Oblique Incidence (TM): Example

$$\tilde{E}^t = \hat{\mathbf{y}} T_{\text{TE}} E_{oy} e^{-j\beta_2\left(x\sin\left(\theta^t\right)+z\cos\left(\theta_t\right)\right)}$$

$$\tilde{H}^t = \frac{T_{\text{TE}} E_{oy}}{\eta_2}\left(\hat{\mathbf{x}}\cos\left(\theta^t\right)+\hat{\mathbf{z}}\sin\left(\theta^t\right)\right) e^{-j\beta_2\left(x\sin\left(\theta^t\right)+z\cos\left(\theta_t\right)\right)}$$