ECE343 Spring 2025

Objective 9.23:

UPW

Characteristics



SCHEDULE

Lesson	Topics	Reading	Notes
24	BVP (Capacitor, Coax), Electric Potential	5.16	LO 5.3
25	Electric Boundary Conditions	5.18-5.22	LO 5.4
26	BC's/Capacitance	5.23-5.24	
27	Electrostatic Lab	Lab packet	
Chapter 6: Steady Current & Conductivity			
28	Current density, Conductivity, Resistance	Ch. 6	
Chapter 7: Magnetostatics			
29	Magnetic Fields, Gauss' Law for Magnetism	7.1-7.3	LO 7.1-7.2 **GR 2**
30	Ampere's Law	7.4-7.5, 7.9	LO 7.3
31	Ampere's Law Applied	7.6-7.8	LO 7.3
32	Magnetic Boundary Conditions	7.10-7.14	LO 7.4
33	Magnetostatics Lab	Lab packet	
Chapter 8: Time-Varying Fields			
34	Faraday's Law, Displacement Current, Maxwell Eqns	8.3, 8.8, 8.9	LO 8.1
35	Faraday's Law, Displacement Current, Maxwell Eqns	8.3, 8.8, 8.9	LO 8.2-8.3
Chapter 9: Plane Waves in Lossless Media			
36	Maxwell's Equations in lossless, source-free region	9.1-9.2	LO 9.1
37	Maxwell's Equations in lossless, source-free region	9.1-9.2	LO 9.1
38	Uniform Plane Waves & Polarization	9.4-9.6	LO 9.2-9.3
39	Normal Incidence	Vol 2, 5.6-5.7	LO 9.4-9.5
40	Oblique Incidence	Vol 2, 5.6-5.7	LO 9.5
41	Oblique Incidence	Vol 2, 5.6-5.7	LO 9.5
42	Incidence Lab	TBD	LO 9.5

Ch 9 Learning Objectives

9.1 I understand the process of decoupling and finding solutions for Maxwell's equations for dynamic fields.

9.2 I can articulate what a Uniform Plane Wave (UPW) is and describe key properties that relate the electric and magnetic field.

9.3 I can calculate the electric and magnetic fields for a UPW.

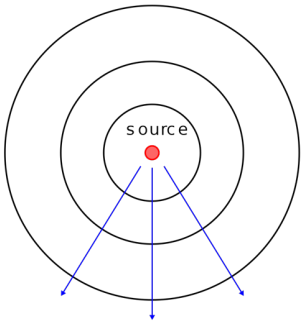
9.4 I can articulate the concept of polarization and determine the polarization for various UPWs.

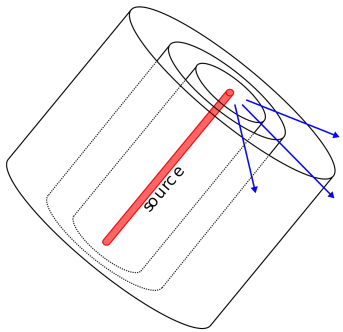
9.5 I can determine the reflection and transmission of UPW at a boundary for Transverse Electric (TE) and Transverse Magnetic (TM) waves, using boundary conditions.

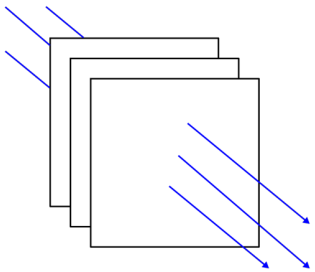
Types of Waves: By Shape of Wave Front

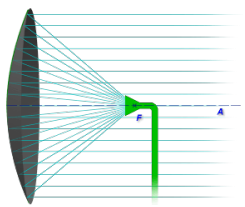
Surfaces of constant phase can be:

- Spherical
  Appropriate when source is small relative to domain size
- Cylindrical
  Appropriate when source is shaped like a line.
- Planar
  Appropriate when wave is observed locally or when wave is intentionally guided to be planar.









Uniform Plane Waves

Assumptions:

- Simple Media:
  - Linear
  - Isotropic
  - Homogeneous
  - Non-dispersive
- Magnitude and phase are constant in a plane perpendicular to direction of propagation (assume z)

$$\frac{\partial \tilde{E}}{\partial x} = \frac{\partial \tilde{E}}{\partial y} = \frac{\partial \tilde{H}}{\partial x} = \frac{\partial \tilde{H}}{\partial y} = 0$$

Which means:

$$\frac{\partial}{\partial x}\tilde{E}_x = \frac{\partial}{\partial x}\tilde{E}_y = \frac{\partial}{\partial x}\tilde{E}_z = 0$$
$$\frac{\partial}{\partial x}\tilde{H}_x = \frac{\partial}{\partial x}\tilde{H}_y = \frac{\partial}{\partial x}\tilde{H}_z = 0$$

Now, the wave equation can be written as:

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right]\left(\hat{\mathbf{x}}\tilde{E}_x + \hat{\mathbf{y}}\tilde{E}_y + \hat{\mathbf{z}}\tilde{E}_z\right)$$
$$+ \gamma^2\left(\hat{\mathbf{x}}\tilde{E}_x + \hat{\mathbf{y}}\tilde{E}_y + \hat{\mathbf{z}}\tilde{E}_z\right) = 0$$

Uniform Plane Waves: Fields

Each component separately must equal zero, allowing us to write 3 equations:

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right]\tilde{E}_{x,y,z}+\gamma^2\tilde{E}_{x,y,z}=0$$

So, the UPW conditions reduce these equations to:

$$\frac{\partial^2}{\partial z^2}\tilde{E}_{x,y,z}+\gamma^2\tilde{E}_{x,y,z}$$

We can make one more simplification, by considering Ampere's Law:

$$\nabla\times\tilde{H}=(\sigma+j\omega\varepsilon)\,\tilde{E}$$

$$\hat{\mathbf{z}}\cdot\left(\nabla\times\tilde{H}\right)=\hat{\mathbf{z}}\cdot\left[(\sigma+j\omega\varepsilon)\,\tilde{E}\right]$$

$$\frac{\partial\tilde{H}_x}{\partial y}-\frac{\partial\tilde{H}_y}{\partial x}=(\sigma+j\omega\varepsilon)\,\tilde{E}_z$$

$$0-0=(\sigma+j\omega\varepsilon)\,\tilde{E}_z$$

$$0=\tilde{E}_z$$

$$\frac{\partial^2}{\partial z^2}\tilde{E}_{x,y}+\gamma^2\tilde{E}_{x,y}$$

Uniform Plane Waves: Fields

We know how to find solutions for the 2nd order ODEs (right?!)

$$\frac{\partial^2}{\partial z^2}\tilde{E}_x + \gamma^2 \tilde{E}_x = 0$$

$$\Longrightarrow \quad \tilde{E}_x(z) = E_{x0}^{+} e^{-j\gamma z} + E_{x0}^{-} e^{j\gamma z}$$

$$\frac{\partial^2}{\partial z^2}\tilde{E}_y + \gamma^2 \tilde{E}_y = 0$$

$$\Longrightarrow \quad \tilde{E}_y(z) = E_{y0}^{+} e^{-j\gamma z} + E_{y0}^{-} e^{j\gamma z}$$

$$\tilde{E} = \hat{\mathbf{x}}\tilde{E}_x + \hat{\mathbf{y}}\tilde{E}_y$$

$$\tilde{H}_x(z) = H_{x0}^{+} e^{-j\gamma z} + H_{x0}^{-} e^{j\gamma z}$$

$$\tilde{H}_y(z) = H_{y0}^{+} e^{-j\gamma z} + H_{y0}^{-} e^{j\gamma z}$$

$$\tilde{H} = \hat{\mathbf{x}}\tilde{H}_x + \hat{\mathbf{y}}\tilde{H}_y$$

$$\gamma = \alpha + j\beta = \sqrt{j\omega\mu\left(\sigma + j\omega\varepsilon\right)}$$

Now, back to time domain...multiply by $e^{j\omega t}$ and take the real part.

$$E_{x,y}(z,t) = \mathrm{Re}\left\{ E_{x0,y0}^{+} e^{-j\beta z} e^{j\omega t} + E_{x0,y0}^{-} e^{j\beta z} e^{j\omega t} \right\}$$

$$E_{x,y}(z,t) = E_{x0,y0}^{+} e^{-\alpha z} \cos\left(\omega t - \beta z\right) + E_{x0,y0}^{-} e^{\alpha z} \cos\left(\omega t + \beta z\right)$$

UPW Summary

Maxwell's Equations

$$\text{Gauss} \longrightarrow \nabla \cdot \tilde{E} = \frac{\rho_v}{\varepsilon}$$

$$\text{Gauss} \longrightarrow \nabla \cdot \tilde{H} = 0$$

$$\text{Ampere} \longrightarrow \nabla \times \tilde{H} = (\sigma + j\omega\varepsilon)\,\tilde{E}$$

$$\text{Faraday} \longrightarrow \nabla \times \tilde{E} = -j\omega\mu\tilde{H}$$

Helmholtz

$$\nabla^2\tilde{E} - \gamma^2\tilde{E} = 0$$

$$\nabla^2\tilde{H} - \gamma^2\tilde{H} = 0$$

$$\gamma^2 = j\omega\mu\,(\sigma + j\omega\varepsilon)$$

Field Solutions

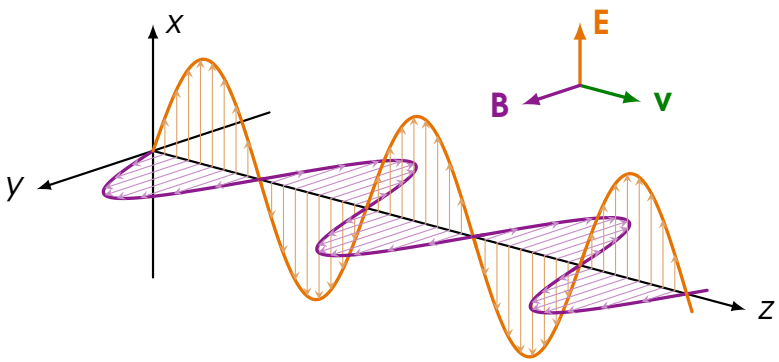
$$E_{x,y}(z,t) = E^{+}_{x0,y0}e^{-\alpha z}\cos(\omega t - \beta z) + E^{-}_{x0,y0}e^{\alpha z}\cos(\omega t + \beta z)$$

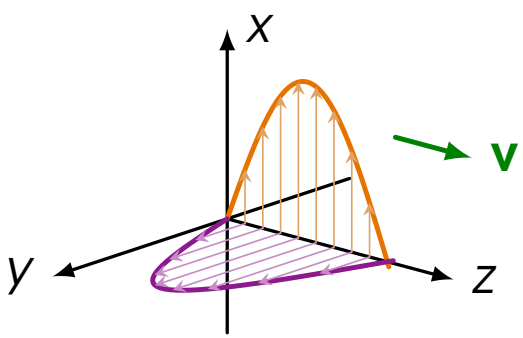
$$H_{x,y}(z,t) = H^{+}_{x0,y0}e^{-\alpha z}\cos(\omega t - \beta z) + H^{-}_{x0,y0}e^{\alpha z}\cos(\omega t + \beta z)$$

Uniform Plane Waves: Interpretation

Observations

- For a UPW, the electric and magnetic field vectors are perpendicular to propagation.
- The direction of propagation is perpendicular to the electric field and magnetic field
- The direction of the electric field and magnetic field are related

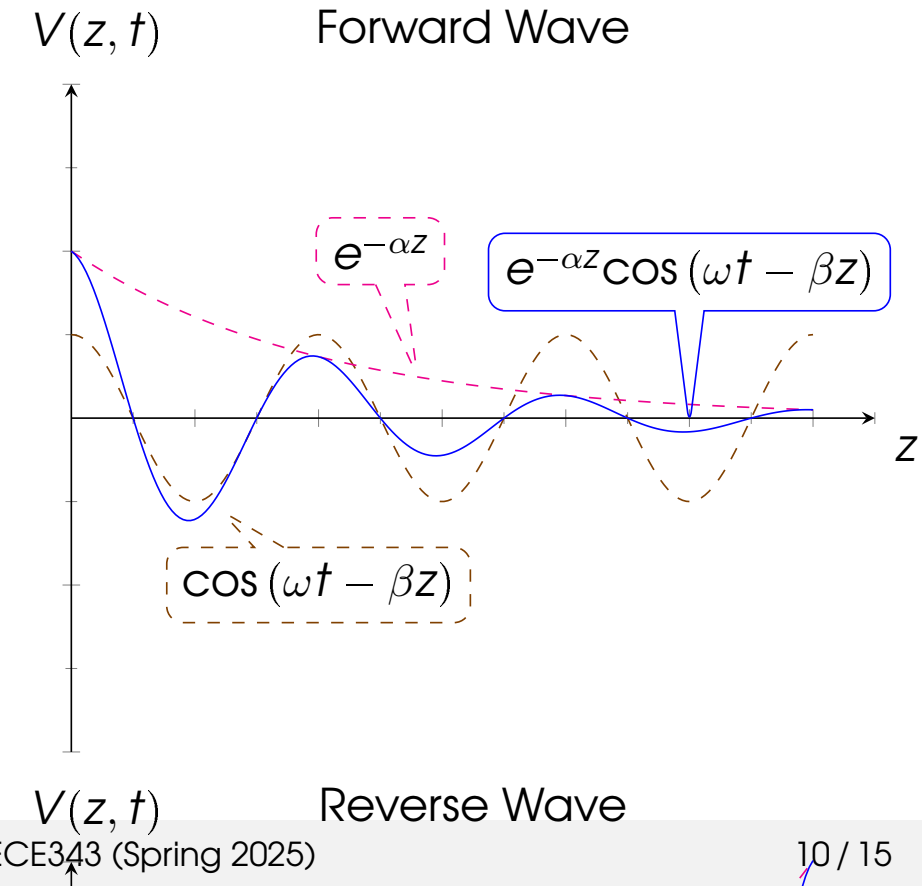




Chapter 3: Lesson 5

$$V(z,t) = \underbrace{V_0^+ e^{-\alpha z}\cos(\omega t - \beta z)}_{\text{Forward Wave}} + \underbrace{V_0^- e^{\alpha z}\cos(\omega t + \beta z)}_{\text{Reverse Wave}}$$

- $V_0^{\pm} \longrightarrow$ Amplitude coefficients
- $e^{\pm \alpha z} \longrightarrow$ Attenuation term
- $\cos(\omega t \pm \beta z) \longrightarrow$Time and space-varying wave



Uniform Plane Waves: Characteristics

Assume an $\hat{\mathbf{x}}$ Electric field...

$$\tilde{E}(z) = \hat{\mathbf{x}}\tilde{E}_x(z)$$

$$\nabla \times \tilde{E} = -j\omega\mu\tilde{H}$$

$$\hat{\mathbf{x}}(0) - \hat{\mathbf{y}}\left(0 - \frac{\partial \tilde{E}_x}{\partial z}\right) + \hat{\mathbf{z}}\left(0 - \frac{\partial \tilde{E}_x}{\partial y}\right)$$

$$\Longrightarrow \hat{\mathbf{y}}\frac{\partial \tilde{E}_x}{\partial z} = -j\omega\mu\tilde{H}$$

$$\frac{\partial \tilde{E}_x}{\partial z} = \frac{\partial}{\partial z}\left(E_{x0}^{+}e^{-\gamma z}\right) + \frac{\partial}{\partial z}\left(E_{x0}^{-}e^{\gamma z}\right)$$

$$\Longrightarrow \hat{\mathbf{y}}\left[-\gamma E_{x0}^{+}e^{-\gamma z} + \gamma E_{x0}^{-}e^{\gamma z}\right] = -j\omega\mu\tilde{H}$$

$$\Longrightarrow \tilde{H} = \hat{\mathbf{y}}\left[\frac{E_{x0}^{+}}{j\omega\mu}e^{-\gamma z} - \frac{E_{x0}^{-}}{j\omega\mu}e^{\gamma z}\right]$$

Intrinsic impedance is like characteristic impedance...

$$\eta = \frac{E_{x0}^{+}}{H_{y0}^{+}} = \frac{E_{x0}^{+}}{\frac{E_{x0}^{+}}{j\omega\mu}} = \frac{j\omega\mu}{\gamma}$$

$$= \frac{j\omega\mu}{\sqrt{j\omega\mu\left(\sigma + j\omega\varepsilon\right)}} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

$\tilde{E}$, $\tilde{H}$, and propagation ($\hat{a}_p$) are **generally** related:

$$\tilde{E} = -\eta\hat{a}_p \times \tilde{H}$$

$$\tilde{H} = \frac{1}{\eta}\hat{a}_p \times \tilde{E}$$

Uniform Plane Waves: General

Summary

$$\vec{E}(z,t) = \hat{\mathbf{x}}E_x(z,t) + \hat{\mathbf{y}}E_y(z,t)$$

$$E_{x,y}(z,t) = E^{\pm}_{x0,y0}e^{\mp\alpha z}\cos(\omega t \mp \beta z)$$

$$\tilde{H}(z,t) = \hat{\mathbf{x}}H_x(z,t) + \hat{\mathbf{y}}H_y(z,t)$$

$$H_{x,y}(z,t) = H^{\pm}_{x0,y0}e^{\mp\alpha z}\cos(\omega t \mp \beta z)$$

$$\tilde{E} = -\eta\hat{a}_p \times \tilde{H} \qquad \tilde{H} = \frac{1}{\eta}\hat{a}_p \times \tilde{E} \qquad \eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

Uniform Plane Waves: Dielectrics

For dielectrics ($\sigma = 0, \varepsilon = \varepsilon_r\varepsilon_0, \mu = \mu_0$):

$$\begin{aligned}\gamma &= \sqrt{j\omega\mu_0\left(0 + j\omega\varepsilon_r\varepsilon_0\right)} \\ &= \sqrt{-\omega^2\mu_0\varepsilon_r\varepsilon_0} \\ &= j\omega\sqrt{\mu_0\varepsilon_r\varepsilon_0} = \frac{j\omega\sqrt{\varepsilon_r}}{c} \\ &= \alpha + j\beta\end{aligned}$$

$$\eta = \frac{j\omega\mu_0}{0 + j\omega\varepsilon_r\varepsilon_0} = \sqrt{\frac{\mu_0}{\varepsilon_r\varepsilon_0}} = \frac{120\pi}{\sqrt{\varepsilon_r}}$$

$$u_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu_0\varepsilon_0\varepsilon_r}} = \frac{c}{\sqrt{\varepsilon_r}}$$

Free Space

$$\eta = \frac{120\pi}{\sqrt{1}} = 120\pi = 377\Omega$$

$$u_p = \frac{c}{\sqrt{\varepsilon_r}} = \frac{c}{\sqrt{1}} = c$$

Silicon, $\varepsilon_r = 11.6$

$$\eta = \frac{120\pi}{\sqrt{11.6}} = 110.7\Omega$$

$$u_p = \frac{c}{\sqrt{11.6}} = 0.3c$$

Uniform Plane Waves: Conductors

For conductors:

$$(\sigma >> \omega\varepsilon) \text{ or } \left(\frac{\sigma}{\omega\varepsilon} >> 1\right)$$

$$\gamma = \sqrt{j\omega\mu\left(\sigma + j\omega\varepsilon_r\varepsilon_0\right)} \approx \sqrt{j\omega\mu\sigma}$$

$$= e^{j\pi/4}\sqrt{\omega\mu\sigma}$$

$$= \left(\cos\left(\frac{\pi}{4}\right) + j\sin\left(\frac{\pi}{4}\right)\right)\sqrt{\omega\mu\sigma}$$

$$\Longrightarrow \alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\eta = \frac{j\omega\mu}{\sigma + j\omega\varepsilon} = \sqrt{\frac{j\omega\mu}{\sigma}}$$

$$= \sqrt{j}\sqrt{\frac{\omega\mu}{\sigma}} = \left(e^{j\pi/2}\right)^{1/2}\sqrt{\frac{\omega\mu}{\sigma}}$$

$$= e^{j\pi/4}\sqrt{\frac{\omega\mu}{\sigma}}$$

What does this mean?!?!

Uniform Plane Waves: Conductors

$\tilde{E} \overset{?}{\leftrightarrow} \tilde{H}$

$$\eta = \frac{E_0^+}{H_0^+} \implies E_0^+ = \eta H_0^+$$

$$\implies E_0^+ = H_0^+ \underbrace{e^{j\pi/4}}_{\text{phase shift}} \sqrt{\frac{\omega\mu}{\sigma}}$$

Dispersion

$$u_p = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\frac{\omega\mu\sigma}{2}}} = \sqrt{\frac{2\omega}{\mu\sigma}}$$

Skin Depth $\sim e^{-\alpha z}$

Define δ as skin depth, where $\tilde{E}(z = \delta) = E_0 e^{-1}$:

$$\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}}$$